Shared Control Individuals in Health Policy Evaluations with Application to Medical Cannabis Laws

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Abstract

Health policy researchers often have questions about the effects of a policy implemented at some cluster-level unit, e.g., states, counties, hospitals, etc. on individual-level outcomes collected over multiple time periods. Stacked difference-in-differences is an increasingly popular way to estimate these effects. This approach involves estimating treatment effects for each policy-implementing unit, then, if scientifically appropriate, aggregating them to an average effect estimate. However, when individual-level data are available and non-implementing units are used as comparators for multiple policy-implementing units, data from untreated individuals may be used across multiple analyses, thereby inducing correlation between effect estimates. Existing methods do not quantify or account for this sharing of controls. Here, we describe a stacked difference-in-differences study investigating the effects of state medical cannabis laws on treatment for chronic pain management that motivated this work, discuss a framework for estimating and managing this correlation due to shared control individuals, and show how accounting for it affects the substantive results.

Keywords: Difference in differences, causal inference, correlated data, insurance claims data, multilevel data

1 Introduction

In 2021, the Centers for Disease Control and Prevention estimated that opioid-related overdoses claimed over 80,000 lives in the United States (Centers for Disease Control and Prevention 2022). The cannabis industry and advocates have argued that state medical cannabis laws could provide a partial solution to this crisis by introducing an alternative treatment for chronic non-cancer pain, an important driver of opioid prescribing in the U.S. (National Cannabis Industry Association 2023). Treatment guidelines for chronic non-cancer pain, which is commonly related to low back pain, fibromyalgia, chronic headaches (including migraine), arthritis, and neuropathic pain, have deemphasized prescription opioids as first-line treatments in recent years. However, these guidelines do not recommend cannabis; rather, they emphasize treatments like non-opioid prescription analgesic medications or procedures (e.g., physical therapy) (Dowell et al. 2022). Despite this, individuals with chronic non-cancer pain are eligible to use medical cannabis for pain management under all existing state medical cannabis laws in the U.S. (National Conference of State Legislatures 2023).

While recent survey evidence shows that chronic non-cancer pain patients report substituting cannabis for prescription opioids (Bicket et al. 2023), empirical policy evaluations have found mixed results on the effects of these laws on opioid prescribing (Shah et al. 2019, Raji et al. 2019, Bradford & Bradford 2016, Bradford et al. 2018, Wen & Hockenberry 2018, Liang et al. 2018, Powell et al. 2018, Bachhuber et al. 2014). However, these empirical studies have largely used general-population and/or repeated cross-sectional samples, which means they consist mostly of individuals without chronic non-cancer pain diagnoses (who are likely unaffected by the law) and/or are unable to follow individuals over time to observe changes in pain treatment. In this work, we describe and reanalyze individual-level data from a study estimating the effect of state medical cannabis laws on opioid prescribing and non-opioid chronic pain treatment receipt for individuals with chronic non-cancer pain (McGinty et al. 2023).

There are a variety of statistical methods that, under assumptions, enable inference for the causal effect of a policy on outcomes of interest relative to a well-defined comparison condition. Broadly, the class of methods that use changes over time across a set of treated and comparison units to estimate effects are referred to as comparative interrupted time series or difference-in-differences (DiD). DiD is a popular approach for estimating policy effects that compares the change over time in an outcome between treated and comparison groups, essentially using the trends in the comparison groups as a proxy for how outcomes would have evolved in the treated group in the absence of the policy change (Wing et al. 2018). The inclusion of a comparison group allows investigators to control for underlying secular trends that might affect both groups, making DiD a stronger design for causal inference than "uncontrolled" approaches that do not include a comparison group and simply compare trends over time in a set of treated unit(s) (Stuart et al. 2014).

Traditionally, the two-way fixed effects (TWFE) model was widely used to estimate the average treatment effect among the treated (ATT) in DiD studies for policy evaluation. In its simplest form, TWFE DiD regresses an outcome on fixed effects for unit and time and an indicator for whether a unit has been treated by that time. Many recent advances in DiD methods aim to solve

problems with TWFE, specifically that it can yield a biased estimate of the treatment effect on average over policy-implementing units when those units implement the policy at different times ("staggered adoption") and the effect of the policy is time-varying (Goodman-Bacon 2021). One way to circumvent problems with TWFE under staggered adoption is to use DiD to estimate separate treatment effects for each policy-implementing unit, then combine those effect estimates for an overall estimate. This approach is commonly known as "stacking" and is related to the idea of serial "trial emulation" (Ben-Michael, Feller & Stuart 2021): effectively, stacking involves emulating a target trial for each treated unit, then pooling effect estimates.

In general, stacking proceeds by identifying, for each treated unit, a "time 0" at which the policy is implemented. Then, a suitable pool of comparison units is identified, and their time 0 is defined to be the same as the treated unit's. These comparators are identified in a principled way; for example, all units that do not implement the policy of interest in a 7-year window around the treated unit's time 0. When control units are chosen in such a way, it is likely that some will be used as comparators for multiple treated units. In the remainder of the article, we often refer to the cluster-level units that may have implemented the policy of interest as "states", though they could also be, e.g., counties, nations, hospitals, etc.

When individual-level data is used in stacked DiD, and when the pool of comparison states is not distinct for each treated state, it is likely that individuals in comparison states may contribute to effect estimation for multiple treated states. These "shared control individuals" meet eligibility criteria for multiple DiDs, and therefore induce correlation between those effect estimates. In the medical cannabis laws study, eligibility criteria were a qualifying diagnosis in a treated state's pre-law period and continuous presence in the health insurance claims database from which the data were collected; we provide more details in Section 2. When aggregating per-state effect estimates, as is often the goal, correlation induced by shared control individuals must be accounted for in order to produce correct inference. This problem may arise in any policy evaluation that uses individual-level data with at least partial sharing of control individuals across time and trial emulations. Our methodological contribution is a procedure to, when individual-level data is available, estimate the correlation between treatment effect estimates that is induced by shared control individuals and to account for it when estimating the variance of the aggregated effect estimate.

We start by describing the quantitative portion of the mixed-methods study that motivates this work and that investigates the effects of state medical cannabis laws on opioid and non-opioid prescribing for individuals with chronic non-cancer pain (McGinty et al. 2021, 2023). We then provide a brief review of DiD in Section 3, introduce an approach that estimates and adjusts for the correlation across stacked DiD estimates in Section 4, and reanalyze data from the medical cannabis laws study in Section 5.

2 Motivation: State Medical Cannabis Laws and Opioid Prescribing

The motivating example for this work is a study designed to estimate the effect of state medical cannabis laws on opioid and guideline-concordant non-opioid prescribing for chronic non-cancer pain treatment among commercially-insured U.S. adults (McGinty et al. 2021, 2023). The study identified a set of 12 "treated" states that enacted a medical cannabis law between 2012 and 2019 and did not also enact a recreational cannabis law within 4 years pre- or 3 years post-cannabis law implementation (CT, MN, NY, NH, FL, MD, PA, OK, OH, ND, AK, LA) and 17 control states (AL, GA, ID, IN, IA, KS, KY, MS, NE, NC, SC, SD, TN, TX, VA, WI, WY) that did not enact medical or recreational cannabis laws over the same period. The primary scientific question asked about the effect of implementing a medical cannabis law on chronic pain treatment outcomes, relative to what would have happened in the absence of a law, on average among the states that implemented such a law.

Outcomes of interest were measures of opioid and guideline-concordant non-opioid prescribing and chronic pain procedures among individuals with chronic non-cancer pain. Importantly, the study did not use a general-population sample, as access to medical cannabis is restricted by state laws to only individuals with a qualifying diagnosis (National Conference of State Legislatures 2023); therefore, individuals without chronic non-cancer pain were excluded from the sample. Opioid-related outcomes studied include receipt of any opioid prescription in a given month, the number of opioid prescriptions among individuals who received at least one, the total morphine milligram equivalents (MME) per day for those prescriptions, the number of days' supply, receipt of an opioid prescription with more than 7 days' supply, and receipt of more than 50 MME per day. The last two outcomes are indicators of high-risk opioid prescribing that increases risk of overdose (Dowell et al. 2022, 2016). Guideline-concordant non-opioid outcomes included receipt of any non-opioid analgesic prescription, the number of such prescriptions among those who received at least one, and receipt and number of treatment(s) via procedures (e.g., surgeries) (McGinty et al. 2023).

This study used de-identified administrative claims data from the Optum Labs Data Warehouse (OLDW), which includes medical and pharmacy claims, laboratory results, and enrollment records for commercial and Medicare Advantage enrollees. The database contains longitudinal health information for over 200 million individuals, representing a mixture of ages and geographic regions across the United States (Optum Labs 2022). The study sample included individuals who reside in a treated or control state, who were continuously enrolled in a commercial or Medicare Advantage plan that provides claims data to OLDW, and who meet eligibility criteria discussed below. Analytic data sets were constructed at the patient-month level. Monthly data allowed for the capture of guideline-concordant prescribing for chronic non-cancer pain, which is often given 30 days at a time.

Each medical cannabis state's law implementation date was defined as the first day of the month in which the state's first medical cannabis dispensary opened. All 12 states had unique implementation dates; therefore, using a traditional TWFE DiD approach may have led to a biased

estimate of the average treatment effect among the treated (ATT). In this study, stacked DiD was used to solve this problem. For each treated state, a unique 7-year study period was constructed, centered around that state's medical cannabis law implementation date: data was collected for 4 years pre- and 3 years post-implementation. The study periods for all 12 treated states are depicted visually in Figure 1.

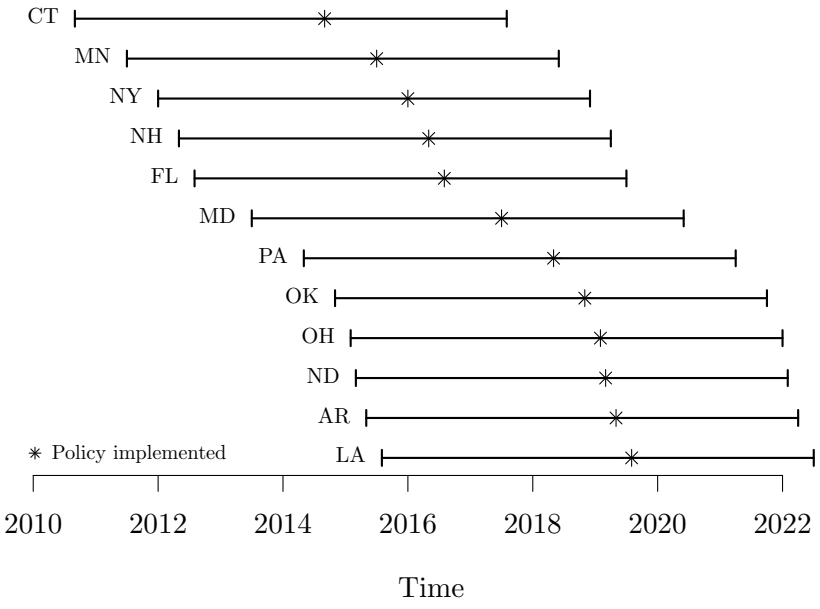

Figure 1: Diagram of study periods for each of the 12 treated states in the medical cannabis study. Stars (*) represent the date on which the state's medical cannabis policy went into effect.

The key idea of stacking is to anchor time for the treated and comparison states, given unique study periods for each treated state, relative to the treated state's implementation date (i.e., set policy implementation as time 0), estimate a treatment effect for each treated state, then aggregate those estimates across all treated states. We describe the specific estimands in more detail in Section 3. Each state-specific effect estimate was based on a treated state-specific dataset that we refer to as a *cohort*. A treated state's cohort consisted of monthly measurements from individuals in the treated state of interest or one of the 17 control states who had at least two insurance claims, on different days, with a primary ICD-9/ICD-10 diagnosis code related to one or more of low back pain, fibromyalgia, chronic headaches (including migraine), arthritis (including rheumatoid and osteoarthritis), or neuropathic pain — conditions commonly leading to chronic non-cancer pain – in the 4 years prior to law implementation *and* who were continuously enrolled in insurance and therefore present in the OLDW data for the entire 7-year study period. Since each cohort contained information from only one treated state, with clear time anchoring at the time of the policy start date for the treated state, TWFE DiD could be used to estimate the treatment effect.

We highlight two important considerations that went into this stacked design. First, using a

common pool of "never-treated" control states avoided difficulties in interpretation and comparison of effect estimates that may arise when the pool of control individuals changes from treated state to treated state, as would be the case if using control states that are "not yet treated". Next, individuals were required to be continuously present in the data over a treated state's 7-year study period to contribute to that state's analysis. This is a key difference between this application and those that motivate other recent DiD advances, which typically assume group-panel data (e.g., annual homicide rates for an entire state) (Roth et al. 2023). In this context, and in many others, this data structure is not available: this study used insurance claims data that individuals may enter and exit over time as they enroll or disenroll from an insurance plan. Requiring individuals be continuously enrolled over the study period minimized bias and challenges in interpretation due to changing composition (of either the treated or control group) over time; this is analogous to active efforts to retain participants over time in a randomized trial.

Imposing a continuous enrollment requirement came at the cost of generalizability: individuals enrolled in a commercial health insurance plan for 7 years are different from those who may be enrolled for only 1-2 years at a time. However, in this case it is highly unlikely that selection into the sample (i.e., whether an individual is continuously enrolled in commercial health insurance over a particular 7-year study period) was related to a state's implementation of a medical cannabis policy, so concerns about bias due to selection on post-treatment characteristics are minimal.

These design considerations, combined with the use of individual-level data, led to (partial) sharing of individuals in control states between cohorts. To see this, consider Figure 2 and the cohorts constructed for CT and MN's analyses. CT implemented its medical cannabis law in September 2014; therefore, its distinct 7-year study period covered September 2010 through August 2017. MN implemented its law in July 2015; its 7-year study period ran from July 2011 through June 2018. Individuals living in CT or MN were included in the sample if they were continuously enrolled for their state's respective 7-year study period and had an eligible chronic pain diagnosis in their state's pre-law period. Individuals living in one of the 17 control states who were continuously enrolled from September 2010 through August 2017 and who have a qualifying chronic non-cancer pain diagnosis between September 2010 and August 2014 (CT's pre-law period) were included in CT's cohort. The control individuals in MN's cohort include those living in one of the 17 control states who were continuously enrolled from July 2011 through June 2018 with a qualifying diagnosis between July 2011 and July 2015. Of the control individuals in either cohort, those who had a qualifying diagnosis between July 2011 and August 2014 (while both CT and MN are in their pre-law periods) and who were continuously enrolled from September 2010 through June 2018 were shared between both CT and MN's analyses. Across the entire study, 84.3% of control individuals contributed to two or more of the 12 cohorts; 4.2% contributed to all 12.

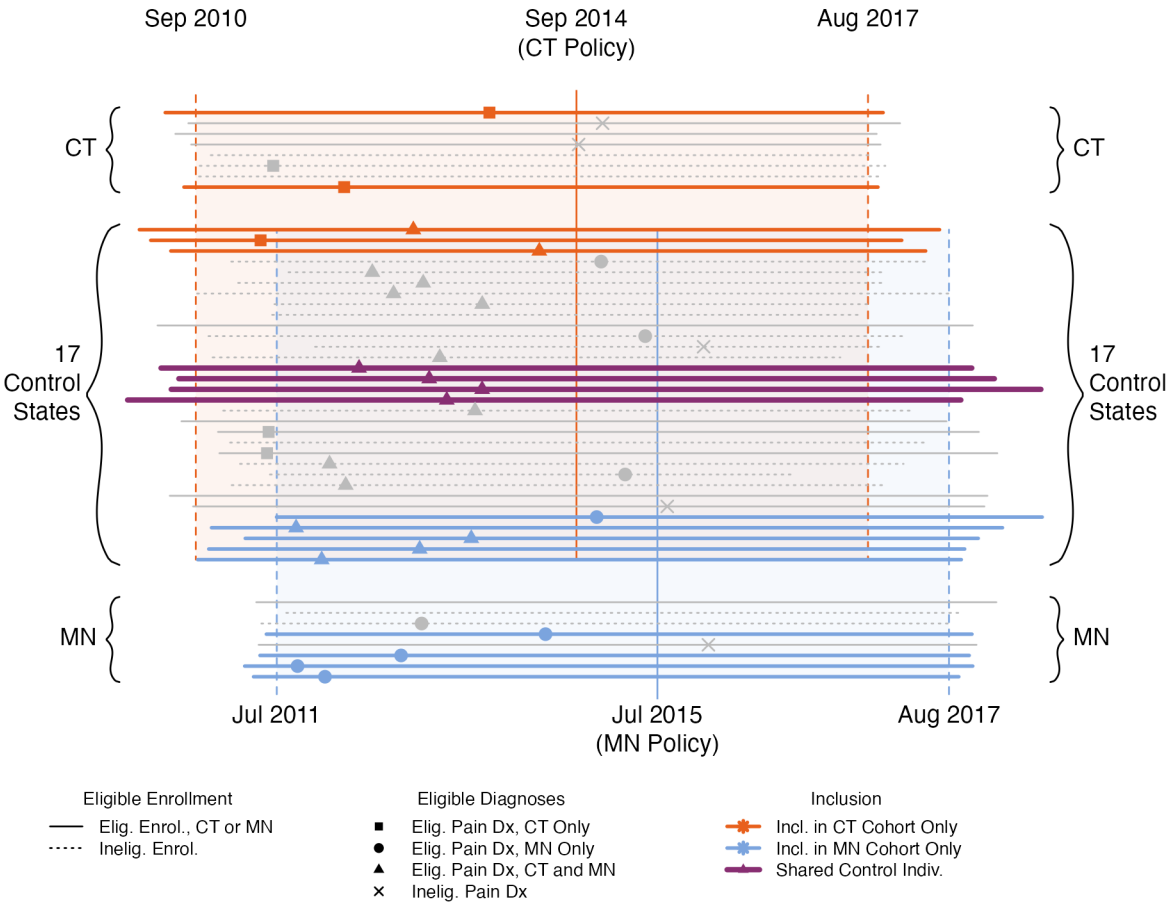

Figure 2: Cohort construction for Connecticut and Minnesota in the medical cannabis laws study. Horizontal lines represent times over which an individual is continuously present in the data: a solid line indicates continuous presence over either CT or MN's full study period (and thus eligibility for inclusion); a dotted line otherwise. The top and bottom groups of lines are individuals in CT and MN, respectively; the middle group individuals in one of the 17 control states. Markers on the lines indicate the time of a qualifying chronic pain diagnosis: squares indicate a diagnosis time allowing inclusion in CT's cohort; circles, only MN's cohort; triangles, both; ×s, neither. Individuals in treated states are eligible for only their states' cohort. Individuals included in a cohort have thick, colored timelines, the color of which indicates the cohort. Very thick purple lines for control individuals indicate inclusion in both cohorts. Ineligible individuals have grey timelines.

3 Difference-in-Differences Estimation

As discussed previously, most of the literature on methods for DiD assume group-panel data, i.e., repeated measures over time at the state level (Roth et al. 2023). The situation, and solution, we describe here is relevant in the wide-range of analyses that use individual-level data on particular populations of interest, aggregated up to some unit/time level (e.g., state-months).

The target estimand in DiD is the average treatment effect among the treated (ATT), the expected difference in potential outcomes under treatment and control conditioned on being treated. There are a number of forms of the ATT that may be of interest, including at a particular time in the post-treatment period or averaged across time and/or states, with a variety of state weights possible (e.g., states weighted equally or by population size). In the medical cannabis laws study, primary scientific interest is in the ATT averaged across states and months over the 3-year period following law implementation. We define this as

$$\text{ATT} = \text{E}\left[\bar{Y}_{\{t \geq t_*\}}(1) - \bar{Y}_{\{t \geq t_*\}}(0) \mid A = 1\right], \tag{1}$$

where $Y(a)$ is the potential outcome under treatment status $a \in \{0, 1\}$, where $a = 1$ indicates treatment and 0 otherwise, and A is the random variable indicating whether or not a state is ever treated; the overbar and subscript $\{t \geq t_*\}$ indicates averaging over the post-treatment period (such that t_* is the policy implementation date; see below for more detail). Note that this is not a state-specific quantity: the expectation is over all treated states. We make three assumptions to identify the ATT from observed data:

A1 *No anticipation.* Potential outcomes prior to treatment are unaffected by treatment, i.e., $Y_t(1) = Y_t(0) = Y_t$ for t in the pre-treatment period ($t < t_*$).

A2 *Consistency.* Observed outcomes are equal to the corresponding potential outcomes under observed post-period treatment status, i.e., $Y_t = AY_t(1) + (1 - A)Y_t(0)$ for $t \geq t^*$.

A3 *Parallel counterfactual trends.* In the absence of treatment, the mean outcome evolution in the treated group would be "parallel" to the mean outcome evolution in the control group. To identify the ATT as defined in Equation (1), this requires that $\text{E}\left[\bar{Y}_{\{t \geq t^*\}}(0) - \bar{Y}_{\{t < t^*\}}(0) \mid A = 1\right] = \text{E}\left[\bar{Y}_{\{t \geq t^*\}}(0) - \bar{Y}_{\{t < t^*\}}(0) \mid A = 0\right]$. The specific form of this assumption varies with the choice of estimand (estimating the ATT at a particular time, for example, requires a different form), and might condition on covariates incorporated into an estimator.

For more details on the assumptions, see for example Zeldow & Hatfield (2021). Under identifiability assumptions A1 to A3 and continuing the scenario in which there is one treated state (or all treated states implement the policy simultaneously), we can consistently estimate the ATT using a simple plug-in estimator:

$$\widehat{\text{ATT}} = \left(\bar{Y}_{\text{tx},\{t \geq t_*\}} - \bar{Y}_{\text{tx},\{t < t_*\}}\right) - \left(\bar{Y}_{\text{ctrl},\{t \geq t_*\}} - \bar{Y}_{\text{ctrl},\{t < t_*\}}\right), \tag{2}$$

where, e.g., $\bar{Y}_{\text{ctrl},\{t \geq t_*\}}$ is the average observed outcome over the post-law period and over all control states (Lechner 2011). Alternatively, one could fit a TWFE regression model, given by

$$Y_{\gamma it} = \beta_{0,\gamma} + \beta_{1,t} + \beta_2 A_{\gamma t} + \epsilon_{\gamma it}, \tag{3}$$

where $Y_{\gamma it}$ is the observed outcome for individual i in state γ at time t, $A_{\gamma t}$ is a treatment indicator for whether state γ has implemented the policy (1) or not (0) at or before time t and $\epsilon_{\gamma it}$ is random mean-zero error. Because we are considering a setting with a single treated state and are weighting each post-treatment time point equally, the ordinary least-squares (OLS) estimate of β_2 is equivalent to the simple estimator of ATT in Equation (2).

As discussed above, Goodman-Bacon (2021) showed that the TWFE estimate $\hat{\beta}_2$ of the treatment effect can be severely biased in settings with multiple treated states that implement the treatment at different times, particularly when the true treatment effect is time-varying. A common way to avoid such bias in a setting with multiple treated units when the estimand is an overall ATT averaged across treated units is to use an approach called *stacking*.

3.1 Stacked Difference-in-Differences

Stacked DiD is an estimation strategy for the ATT in which we obtain cohort-specific effect estimates and then pool them, i.e., combine them by taking a (weighted) average. In contrast to a traditional TWFE approach, stacking involves the construction of a series of cohorts, one for each set of states that implemented the policy of interest at the same point in time (at the time level of analysis; e.g., in the same month for state-month level data), with a DiD model run separately for each cohort and then the results averaged together. In Section 2 and Figure 2, we briefly described the construction of state-specific cohorts and how control individuals may be shared across them. Here, we establish notation and formally define shared control individuals in that context.

Consider a set of S states $\Xi = \Xi_{\text{tx}} \cup \Xi_{\text{ctrl}}$ where Ξ_{tx} is the set of S_{tx} treated states, Ξ_{ctrl} the collection of S_{ctrl} control states, and $\Xi_{\text{tx}} \cap \Xi_{\text{ctrl}} = \emptyset$. In the medical cannabis laws study, $S_{\text{tx}} = 12$ and $S_{\text{ctrl}} = 17$. For simplicity of exposition, we proceed as if all treated states implement the policy of interest at different times (i.e., there are S_{tx} cohorts in the stacked DiD), though all results hold if some states implement simultaneously. For a state $\gamma \in \Xi_{\text{tx}}$, define its study period $\mathcal{T}_\gamma = \{t_{1\gamma}, \ldots, t_{*\gamma}, \ldots, t_{T\gamma}\}$ as the set of consecutive measurement occasions at which data is collected, with $t_{*\gamma}$ the first measurement after treatment. We assume the study periods $\mathcal{T}_\gamma$ ($\gamma \in \Xi_{\text{tx}}$) are all of length T with T_{pre} measurements before and T_{post} after treatment. This last condition is a design choice made in the medical cannabis laws study: because scientific interest is in an ATT on average over the post-law period, it was important to maintain identical study period durations.

Every state $\zeta \in \Xi$ is a collection of individuals; say $i \in \zeta$ if individual i lives in state ζ. Individual i is continuously present in the data over U_i consecutive measurement occasions $\mathcal{U}_i = \{u_{1i}, \ldots u_{U_i i}\}$.

For a pair of states $(\gamma, \zeta) \in \Xi_{\text{tx}} \times \Xi$, define

$$\mathcal{I}_\gamma(\zeta) = \{i \in \zeta : i \text{ meets inclusion criteria for analysis of treated state } \gamma\}$$

as the collection of $N_\gamma(\zeta)$ individuals in state ζ that contribute to the analysis for treated state γ. If we impose a continuous presence requirement, the inclusion criteria for individual i include $\mathcal{T}_\gamma \subset \mathcal{U}_i$. Additionally, by design, $\mathcal{I}_\gamma(\zeta) = \emptyset$ for $\zeta \in \Xi_{\text{tx}} / \{\gamma\}$. In the context of the medical cannabis laws study, $\mathcal{I}_{\text{CT}}(\text{AL})$ is the set of all individuals in the data who live in Alabama (a control state), had a chronic non-cancer pain diagnosis in the four years prior to Connecticut's medical cannabis policy implementation, and were continuously enrolled in commercial health insurance over Connecticut'sstudy period. The analytic sample for treated state γ is

$$\mathcal{I}_\gamma = \bigcup_{\zeta \in \Xi} \mathcal{I}_\gamma(\zeta) = \bigcup_{\zeta \in (\{\gamma\} \cup \Xi_{\text{ctrl}})} \mathcal{I}_\gamma(\zeta),$$

the union over γ and control states of individuals in each state who contribute to analysis for state γ.

We define a treated state γ's *cohort* $\mathcal{C}_\gamma$ as the collection of person-time that contributes to estimation of the treatment effect for γ:

$$\mathcal{C}_\gamma = \mathcal{I}_\gamma \times \mathcal{T}_\gamma = \{(i,t) : i \in \mathcal{I}_\gamma, t \in \mathcal{T}_\gamma\}. \tag{4}$$

Using data from a treated unit's cohort, we can estimate ATT_γ, the average treatment effect in treated state γ, using the simple plug-in DiD estimator from Equation (2):

$$\begin{aligned} \widehat{\text{ATT}}_\gamma = & \frac{1}{N_\gamma(\gamma)} \sum_{i \in \mathcal{I}_\gamma(\gamma)} \left(\frac{1}{T_{\text{post}}} \sum_{\{t \geq t_{*\gamma}\}} Y_{\gamma it} - \frac{1}{T_{\text{pre}}} \sum_{\{t < t_{*\gamma}\}} Y_{\gamma it} \right) \\ & - \frac{1}{\sum_{\zeta \in \Xi_{\text{ctrl}}} N_\gamma(\zeta)} \sum_{\zeta \in \Xi_{\text{ctrl}}} \sum_{i \in \mathcal{I}_\gamma(\zeta)} \left(\frac{1}{T_{\text{post}}} \sum_{\{t \geq t_{*\gamma}\}} Y_{\zeta it} - \frac{1}{T_{\text{pre}}} \sum_{\{t < t_{*\gamma}\}} Y_{\zeta it} \right). \end{aligned} \tag{5}$$

As before, this estimator is equivalent to β_2 in a TWFE regression as specfied by Equation (3) because there is only a single treated state. We proceed with a focus on this particular estimator for ATT_γ, though the correlation results are assumed to extend and can be applied to other estimators as well.

The final step of a stacked DiD analysis is to pool the $\widehat{\text{ATT}}_\gamma$s. In a setting without shared control individuals (i.e., when all effect estimates are uncorrelated), a natural way to aggregate might be to use an inverse-variance weighted average, which places more weight on more precise estimates and is the minimum-variance aggregation strategy for uncorrelated estimates (Hartung et al. 2008, ch. 4):

$$\widehat{\text{ATT}} = \frac{1}{\sum_{\gamma \in \Xi_{\text{tx}}} 1/v_{\gamma\gamma}} \sum_{\gamma \in \Xi_{\text{tx}}} \frac{1}{v_{\gamma\gamma}} \widehat{\text{ATT}}_\gamma, \tag{6}$$

where $v_{\gamma\gamma} = \text{Var}\left(\widehat{\text{ATT}}_\gamma\right)$. In a setting with shared control individuals, and thus correlated estimates, we propose a using different weighted average arising from generalized least-squares that incorporates non-zero covariance between estimates. We discuss this in more detail in Section 4.

In practice, researchers who estimate the ATT using TWFE will typically adjust the OLS estimate of the standard error of $\hat{\beta}_2$ in Equation (3) to account for clustering of individuals within states. We do not do that here: in settings with one treated state and many control states, Rokicki et al. (2018) found that typical cluster adjustments lead to poor confidence interval coverage and should not be used. Mathematically, our analyses without cluster adjustment are equivalent to Rokicki et al.'s "aggregated" analysis strategy, which does not suffer from undercoverage problems. We defer our approach to estimating the variance of $\widehat{\text{ATT}}$ to Section 4.

3.2 Shared Control Individuals in Stacked Difference-in-Differences

Consider two treated states $\gamma, \nu \in \Xi_{\text{tx}}$. An individual in control unit $\zeta \in \Xi_{\text{ctrl}}$ is *shared* between cohorts C_γ and C_ν if that individual is in the intersection $\mathcal{I}_\gamma(\zeta) \cap \mathcal{I}_\nu(\zeta)$. This sharing occurs as a consequence of inclusion criteria for each cohort; the number of control individuals shared between two cohorts can change based on continuous presence requirements or the amount of time overlap between the cohorts' study periods $\mathcal{T}_\gamma$ and $\mathcal{T}_\nu$.

For any pair of cohorts $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$, we define the following quantities. Let $N_\gamma = \left|\bigcup_{\zeta \in \Xi} \mathcal{I}_\gamma(\zeta)\right|$ be the total number of individuals who contribute data to cohort $\mathcal{C}_\gamma$. For any $\zeta \in \Xi$, we decompose $N_\gamma(\zeta) = |\mathcal{I}_\gamma(\zeta)|$, the number of individuals in state ζ that contribute to cohort $\mathcal{C}_\gamma$, into the sum $N_{\gamma/\nu}(\zeta) + N_{\gamma\cap\nu}(\zeta)$, where $N_{\gamma/\nu}(\zeta)$ is the number of individuals in state ζ who are included in $\mathcal{C}_\gamma$ but not $\mathcal{C}_\nu$ and $N_{\gamma\cap\nu}(\zeta)$ the number of individuals in state ζ who contribute to both $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$. Further define $N_\gamma^{\text{ctrl}} := \sum_{\zeta \in \Xi_{\text{ctrl}}} N_\gamma(\zeta) = N_\gamma - N_\gamma(\gamma)$. Finally, let $\Delta = |t_{*\gamma} - t_{*\nu}|$ be the number of measurement occasions between the implementation dates for treated states γ and ν.

4 Correlation between Estimates due to Shared Control Individuals

In order to understand the correlation between two state-specific treatment effect estimates $\widehat{\text{ATT}}_\gamma$ and $\widehat{\text{ATT}}_\nu$ induced by shared control individuals, we make some simplifying assumptions about the dependence structure of data within a state. Borrowing from the literature on multi-period cluster-randomized trials, we consider three types of correlation: within-person, within-period, and between-period (Kasza et al. 2019). First, within-person (or intra-individual, longitudinal, or serial) correlation occurs when repeated outcome measures are collected on the same individual over time. Similarly, we expect individuals within the same unit to be related to each other: this is between-person correlation, which has two components: we expect observations from different individuals collected at the same time within the same unit to be correlated ("within-period" correlation), as well as observations from different individuals in the same unit at different times ("between-period" correlation). As is common in the policy evaluation methods literature, we assume states

are mutually independent; i.e., that $\text{Cor}(Y_{\gamma it}, Y_{\nu js}) = 0$ for all $\gamma \neq \nu$.

For simplicity, we assume a block exchangeable correlation structure for all states. For two individuals i, j in the same state $\gamma \in \Xi$ and timepoints t and s, $\text{Cor}(Y_{\gamma it}, Y_{\gamma is}) = \rho_\gamma$ (within-person correlation), $\text{Cor}(Y_{\gamma it}, Y_{\gamma jt}) = \phi_\gamma$ (within-period correlation), and $\text{Cor}(Y_{\gamma it}, Y_{\gamma js}) = \psi_\gamma$ (between-period correlation). For a state γ, then, the correlation matrix for all observations is block-diagonal with $\text{Exch}_T(\rho_\gamma)$ correlation matrices on the diagonal and off-diagonal blocks $\psi_\gamma \mathbf{1}_T \mathbf{1}_T^\top + (\phi_\gamma - \psi_\gamma) I_T$, where $\text{Exch}_T(\rho)$ is a $T \times T$ matrix with 1's on the diagonal and all off-diagonal elements are ρ, $\mathbf{1}_T$ is a T-vector of 1's, and I_T is the $T \times T$ identity matrix. This is depicted visually in Equation (7).

$$\Sigma_\gamma := \text{Var}(\boldsymbol{Y}_\gamma) = \begin{pmatrix} 1 & \rho_\gamma & \cdots & \rho_\gamma & & \phi_\gamma & \psi_\gamma & \cdots & \psi_\gamma \\ \rho_\gamma & 1 & \cdots & \rho_\gamma & & \psi_\gamma & \phi_\gamma & \cdots & \psi_\gamma \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \rho_\gamma & \rho_\gamma & \cdots & 1 & & \psi_\gamma & \psi_\gamma & \cdots & \phi_\gamma \\ & & \vdots & & \ddots & & & \vdots & \\ \phi_\gamma & \psi_\gamma & \cdots & \psi_\gamma & & 1 & \rho_\gamma & \cdots & \rho_\gamma \\ \psi_\gamma & \phi_\gamma & \cdots & \psi_\gamma & & \rho_\gamma & 1 & \cdots & \rho_\gamma \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \psi_\gamma & \psi_\gamma & \cdots & \phi_\gamma & & \rho_\gamma & \rho_\gamma & \cdots & 1 \end{pmatrix} \sigma_\gamma^2, \tag{7}$$

with $\sigma_\gamma^2 = \text{Var}(Y_{\gamma it})$ for all i and t.

Under this covariance structure, we can find an analytic form of both the variance of single-unit DiD estimates $\widehat{ATT}_\gamma$ as in Equation (5) and the pairwise covariance between two such estimates; all such derivations are given in Appendix A. For a given treated unit $\gamma \in \Xi_{\text{tx}}$, the variance of $\widehat{\text{ATT}}_\gamma$ is

$$\text{Var}\left(\widehat{\text{ATT}}_\gamma\right) =: v_{\gamma\gamma} = \frac{T}{T_{\text{pre}} T_{\text{post}}} \sum_{\zeta \in (\{\gamma\} \cup \Xi_{\text{ctrl}})} \frac{\sigma_\zeta^2}{\left(N_\gamma^{A_\zeta}\right)^2} \left[N_\gamma(\zeta)(1 - \rho_\zeta) + N_\gamma(\zeta)(N_\gamma(\zeta) - 1)(\phi_\zeta - \psi_\zeta) \right], \tag{8}$$

where $A_\zeta = \mathbb{1}\{\zeta \in \Xi_{\text{tx}}\}$ is an indicator for whether state ζ was ever treated (as in Equation (1)) such that $N_\gamma^{A_\zeta} = A_\zeta N_\gamma^{\text{tx}} + (1 - A_\zeta) N_\gamma^{\text{ctrl}}$. That is, if $\zeta \in \Xi_{\text{ctrl}}$, then $N_\gamma^{A_\zeta} = N_\gamma^{\text{ctrl}}$ is the total number of individuals in control states that contribute to $\widehat{ATT}_\gamma$. Similarly, if $\zeta \in \Xi_{\text{tx}}$ (i.e., $\zeta = \gamma$), then $N_\gamma^{A_\zeta} = N_\gamma^{\text{tx}} = N_\gamma(\gamma)$.

For a treated state $\gamma \in \Xi_{\text{tx}}$, decompose the cohort $\mathcal{C}_\gamma$ as

$$\mathcal{C}_\gamma = \mathcal{C}_\gamma^{\text{tx}} \cup \mathcal{C}_\gamma^{\text{ctrl}} = (\mathcal{I}_\gamma(\gamma) \times \mathcal{T}_\gamma) \cup \left(\bigcup_{\zeta \in \Xi_{\text{ctrl}}} \mathcal{I}_\gamma(\zeta) \times \mathcal{T}_\gamma \right);$$

that is, let $\mathcal{C}_\gamma^{\text{tx}}$ be the collection of person-times from the treated state γ in cohort $\mathcal{C}_\gamma$ and similarly for $\mathcal{C}_\gamma^{\text{ctrl}}$. Consider now any pair of treated states $\gamma, \nu \in \Xi_{\text{tx}}$. Under the assumption that states are

independent, $\mathrm{Cov}\left(\widehat{\mathrm{ATT}}_\gamma, \widehat{\mathrm{ATT}}_\nu\right)$ depends solely on information from control states, as they are the only states to contribute to both estimates:

$$\begin{aligned}\mathrm{Cov}\left(\widehat{\mathrm{ATT}}_\gamma, \widehat{\mathrm{ATT}}_\nu\right) = \mathrm{Cov}\left(\bar{Y}_{\mathcal{C}_\gamma^{\mathrm{ctrl}},\{t\geq t_{*\gamma}\}}, \bar{Y}_{\mathcal{C}_\nu^{\mathrm{ctrl}},\{t\geq t_{*\nu}\}}\right) + \mathrm{Cov}\left(\bar{Y}_{\mathcal{C}_\gamma^{\mathrm{ctrl}},\{t< t_{*\gamma}\}}, \bar{Y}_{\mathcal{C}_\nu^{\mathrm{ctrl}},\{t< t_{*\nu}\}}\right) \\ - \mathrm{Cov}\left(\bar{Y}_{\mathcal{C}_\gamma^{\mathrm{ctrl}},\{t\geq t_{*\gamma}\}}, \bar{Y}_{\mathcal{C}_\nu^{\mathrm{ctrl}},\{t< t_{*\nu}\}}\right) - \mathrm{Cov}\left(\bar{Y}_{\mathcal{C}_\gamma^{\mathrm{ctrl}},\{t< t_{*\gamma}\}}, \bar{Y}_{\mathcal{C}_\nu^{\mathrm{ctrl}},\{t\geq t_{*\nu}\}}\right),\end{aligned} \tag{9}$$

where, e.g.,

$$\bar{Y}_{\mathcal{C}_\gamma^{\mathrm{ctrl}},\{t\geq t_{*\gamma}\}} = \frac{1}{T_{\mathrm{post}} N_\gamma^{\mathrm{ctrl}}} \sum_{\zeta\in\Xi^{\mathrm{ctrl}}} \sum_{i\in\mathcal{I}_\gamma(\zeta)} \sum_{\{t\geq t_{*\gamma}\}} Y_{\zeta it}$$

is the average outcome over all post-treatment periods and all control states.

Again assuming the covariance structure in Equation (7), then

$$\begin{aligned}\mathrm{Cov}\left(\widehat{\mathrm{ATT}}_\gamma, \widehat{\mathrm{ATT}}_\nu\right) = \frac{f\left(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta\right)}{N_\gamma^{\mathrm{ctrl}} N_\nu^{\mathrm{ctrl}}} \sum_{\zeta\in\Xi_{\mathrm{ctrl}}} \sigma_\zeta^2 \left[N_\gamma(\zeta) N_\nu(\zeta)\left(\phi_\zeta - \psi_\zeta\right)\right. \\ \left. + N_{\gamma\cap\nu}(\zeta)\left(1-\rho_\zeta-\left(\phi_\zeta-\psi_\zeta\right)\right)\right],\end{aligned} \tag{10}$$

where

$$\begin{aligned}f\left(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta\right) = \frac{1}{T_{\mathrm{pre}}^2 T_{\mathrm{post}}^2} \cdot \left(T_{\mathrm{pre}}^2 \max\left(T_{\mathrm{post}}-\Delta, 0\right) + T_{\mathrm{post}}^2 \max\left(T_{\mathrm{pre}}-\Delta, 0\right)\right. \\ \left. - T_{\mathrm{pre}} T_{\mathrm{post}} \min\left(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta, \max\left(T_{\mathrm{pre}}+T_{\mathrm{post}}-\Delta, 0\right)\right)\right).\end{aligned} \tag{11}$$

See Appendix A for a full derivation.

The summand in Equation (10) is strictly positive under the reasonable assumption that for any $\zeta \in \Xi$, $\rho_\zeta \geq \phi_\zeta \geq \psi_\zeta$, i.e., that within-person correlation is higher than between-person correlation and that observations among different individuals at the same time are more correlated than observations at different times. However, the subtracted component of the "time factor" $f(\cdot)$ (Equation (11)) suggests that this multiplier on the covariance may not be everywhere-positive. Indeed, $f(\cdot)$ has two zeros in Δ at $\Delta^* := (T_{\mathrm{pre}}^2 T_{\mathrm{post}} + T_{\mathrm{pre}} T_{\mathrm{post}}^2)/(T_{\mathrm{pre}}^2 + T_{\mathrm{pre}} T_{\mathrm{post}} + T_{\mathrm{post}}^2)$ and $\Delta^\dagger := T_{\mathrm{pre}} + T_{\mathrm{post}}$. Note that $\Delta^* < \Delta^\dagger$ for positive T_{pre} and T_{post}. For $\Delta \in \left[\Delta^*, \Delta^\dagger\right]$, $f(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta) \leq 0$, for $\Delta < \Delta^*$, $f(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta) > 0$, and $f(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta) = 0$ for $\Delta > \Delta^\dagger$. The factor $f\left(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta\right)$ thus controls the sign of the covariance in Equation (9), depending on the amount of time overlap between cohorts' study periods. When there is no time overlap, the covariance is exactly zero: no observations are shared between cohorts' DiD analyses.

In the context of the medical cannabis laws study, where $T_{\mathrm{pre}} = 48$ and $T_{\mathrm{post}} = 36$, $\Delta^* = 27.24$ and $\Delta^\dagger = 84$. $f(48, 36, \Delta)$ is nonpositive for $\Delta \in \{28, \ldots, 84\}$ (see Figure 3). Therefore, the DiD estimate for Connecticut, for example, will have positive covariance with the estimates from Minnesota, New York, New Hampshire, and Florida ($\Delta = 10, 16, 20, 23$, respectively) and negative covariance with the estimates from Maryland, Pennsylvania, Oklahoma, Ohio, North Dakota, Arkansas, and Louisiana ($\Delta = 34, 44, 50, 53, 54, 56, 59$, respectively).

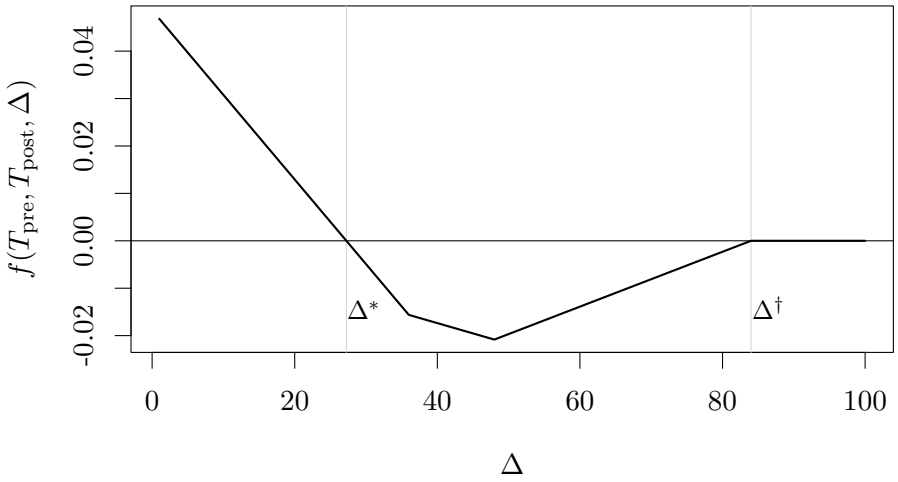

Figure 3: The "time factor" in Equation (9) for $T_{\text{pre}} = 48$ and $T_{\text{post}} = 36$ as in the medical cannabis laws study.

This sign change may be rather surprising: one might expect that allowing information to contribute to multiple effect estimates would induce strictly positive correlation between those estimates. However, this is not always the case in a DiD setting due to the idiosyncratic nature of the DiD estimator and the use of some time periods as pre-policy in one cohort and post-policy in another. These "cross-period" timepoints induce the "cross-period" covariances that are subtracted in Equation (9). As Δ increases from 0, the number of measurement occasions in which cohorts C_γ and C_ν are *both* in their pre-treatment periods (and thus both in their post-treatment periods as well) decreases. When $\Delta > \Delta^*$, this is no longer true: the number of cross-period measurement occasions exceeds those in both cohorts' pre- or post-treatment periods. The number of measurement occasions in a period is a proxy for the amount of information that period contributes to a treatment effect estimate. When the bulk of information shared between two estimates comes from periods in which the treated states have different treatment statuses, this increases the subtracted components of Equation (9).

As an example, consider Maryland and Oklahoma, for which $\Delta_{\text{MD,OK}} = 16$ months. Maryland implemented its medical cannabis law in July 2017; Oklahoma in November 2018. Both states' cohorts are in their pre-law periods for 32 months, from November 2014 through June 2017, and both are in their post-law periods for 20 months, from November 2018 through June 2020. By contrast, there are only 16 months of cross-period time, from July 2017 through October 2018, while Maryland's law was in place and Oklahoma's was not. Thus, the DiD analyses for Maryland and Oklahoma use data from 52 measurement occasions in the same way (i.e., as either pre- or post-law), and data from only 16 measurement occasions are used differently. We would therefore expect positive covariance between the two DiD estimates in this case, and that is the case: $f(48, 36, 16) = 0.02 > 0$. By contrast, for New York and Oklahoma, $\Delta_{\text{NY,OK}} = 34$. Now, there are only 16 months

in which both states are either pre-law (14 months) or post-law (2 months), and 34 cross-period months. Because more information is being used in different ways by the two analyses, we now expect negative covariance between the estimates, and indeed $f(48, 36, 34) = -0.01 < 0$.

Finally, for completeness, we define the correlation between $\widehat{\text{ATT}}_\gamma$ and $\widehat{\text{ATT}}_\nu$ as

$$\text{Cor}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right) = \frac{\text{Cov}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right)}{\text{Var}\left(\widehat{\text{ATT}}_\gamma\right)^{1/2} \text{Var}\left(\widehat{\text{ATT}}_\gamma\right)^{1/2}}, \tag{12}$$

which we estimate with a simple plug-in estimator.

4.1 Aggregation of Correlated Effect Estimates

In the medical cannabis laws study, the estimand of interest was the ATT on average over all 12 treated states. As mentioned above, one strategy for aggregating uncorrelated unit-specific estimates $\{\widehat{\text{ATT}}_\gamma\}$ is inverse-variance weighting (IVW). Define the $S_{\text{tx}} \times S_{\text{tx}}$ diagonal matrix $\boldsymbol{V} = \text{diag}\left(\{v_{\gamma\gamma}\}_{\gamma \in \Xi_{\text{tx}}}\right)$ that has as its diagonal entries the variances of the $\{\widehat{\text{ATT}}_\gamma\}$ as defined in Equation (8). Then the IVW estimate of the overall ATT, as in Equation (6), is

$$\widehat{\text{ATT}}_{\text{ivw}} := \left(\mathbf{1}_{S_{\text{tx}}}^\top \boldsymbol{V}^{-1} \mathbf{1}_{S_{\text{tx}}}\right)^{-1} \mathbf{1}_{S_{\text{tx}}} \boldsymbol{V}^{-1} \widehat{\mathbf{ATT}}_{\Xi_{\text{tx}}} = \frac{1}{\sum_{\gamma \in \Xi_{\text{tx}}} v_{\gamma\gamma}^{-1}} \sum_{\gamma \in \Xi_{\text{tx}}} v_{\gamma\gamma}^{-1} \widehat{\text{ATT}}_\gamma, \tag{13}$$

where $\widehat{\mathbf{ATT}}_{\Xi_{\text{tx}}}$ is the S_{tx}-vector of treated state-specific $\widehat{\text{ATT}}$s. This estimator has variance

$$\text{Var}\left(\widehat{\text{ATT}}_{\text{ivw}}\right) = \left(\mathbf{1}_{S_{\text{tx}}}^\top \boldsymbol{V}^{-1} \mathbf{1}_{S_{\text{tx}}}\right)^{-1} = \frac{1}{\sum_{\gamma \in \Xi_{\text{tx}}} 1/v_{\gamma\gamma}}. \tag{14}$$

The above IVW procedure assumes that the $\{\widehat{\text{ATT}}_\gamma\}$ are uncorrelated. Now define the $S_{\text{tx}} \times S_{\text{tx}}$ matrix $\boldsymbol{W} = \text{Var}\left(\widehat{\mathbf{ATT}}_{\Xi_{\text{tx}}}\right)$ with entries $w_{\gamma\nu} = \text{Cov}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right)$.

We propose an aggregation strategy that accounts for correlated effect estimates by substituting $\boldsymbol{W}$ for $\boldsymbol{V}$ in Equations (13) and (14). This is equivalent to estimating the overall ATT by fitting an intercept-only model with generalized least squares (GLS), following the approach of Lin & Sullivan (2009). Our "GLS-aggregated" estimate $\widehat{\text{ATT}}_{\text{gls}}$ of the overall ATT is the weighted average

$$\widehat{\text{ATT}}_{\text{gls}} = \left(\mathbf{1}_{S_{\text{tx}}}^\top \boldsymbol{W}^{-1} \mathbf{1}_{S_{\text{tx}}}\right)^{-1} \mathbf{1}_{S_{\text{tx}}} \boldsymbol{W}^{-1} \widehat{\mathbf{ATT}}_{\Xi_{\text{tx}}} \tag{15}$$

where $\widehat{\mathbf{ATT}}_{\Xi_{\text{tx}}}$ is the vector of treated state-specific $\widehat{\text{ATT}}$s. This estimator explicitly adjusts for the between-estimate correlation and has variance

$$\text{Var}\left(\widehat{\text{ATT}}_{\text{gls}}\right) = \left(\mathbf{1}_{S_{\text{tx}}}^\top \boldsymbol{W}^{-1} \mathbf{1}_{S_{\text{tx}}}\right)^{-1}. \tag{16}$$

Note that, if $\boldsymbol{W}$ is diagonal (i.e., $\boldsymbol{W} = \boldsymbol{V}$), Equations (13) and (15) are equivalent.

To illustrate the substantive difference between variances (14) and (16), consider a simplified

setting with just two treated units. If the between-estimate correlation is $\kappa = w_{12}/\sqrt{w_{11}w_{22}}$, then $\text{Var}\left(\widehat{\text{ATT}}_{\text{gls}}\right) = \frac{w_{11}+w_{22}-2\kappa\sqrt{w_{11}w_{22}}}{(w_{11}+w_{22})(1-\kappa^2)}$ and $\text{Var}\left(\widehat{\text{ATT}}_{\text{ivw}}\right) = \frac{w_{11}+w_{22}}{w_{11}w_{22}}$. When $0 \leq \kappa \leq \frac{2\sqrt{w_{11}w_{22}}}{w_{11}+w_{22}}$, the GLS-adjusted estimator (15) has *smaller* variance (i.e., is more efficient) than the naïve estimator in equation (13). Otherwise, accounting for between-estimate correlation leads to larger standard errors in the two-cohort setting.

5 Simulations and Reanalysis of Medical Cannabis Laws Study

We designed a simulation study to investigate the properties of Equation (12). We simulate outcomes $Y_{\gamma it}$ from a modified version of model (2) of Kasza et al. (2019) such that

$$Y_{\gamma it} = \beta_0 + \beta_1(t) + \beta_2 A_{\gamma t} + b_{\gamma i} + c_{\gamma t} + \epsilon_{\gamma it}, \tag{17}$$

where $b_{\gamma i} \sim N(0, \sigma^2_{b\gamma})$ is an individual-specific random intercept for person i in state γ, $c_{\gamma t}$ is the t-th element of the T vector of state-time random effects $\boldsymbol{c}_\gamma \sim N_T(\boldsymbol{0}, \sigma^2_{c\gamma}\boldsymbol{R}_\gamma)$ where $\boldsymbol{R}_\gamma$ has (s,t)th element $\text{Cor}(c_{\gamma s}, c_{\gamma t}) = \psi_\gamma/\phi_\gamma$, and $\epsilon_{\gamma it} \sim N(0, \sigma^2_{e\gamma})$ is random error. To achieve the correlation structure in Equation (7), we set $\sigma^2_{b\gamma} = \frac{\rho_\gamma - \psi_\gamma}{(1-\rho_\gamma)-(\phi_\gamma-\psi_\gamma)} \cdot \sigma^2_{e\gamma}$ and $\sigma^2_{c\gamma} = \frac{\phi_\gamma}{(1-\rho_\gamma)-(\phi_\gamma-\psi_\gamma)} \cdot \sigma^2_{e\gamma}$; $\sigma^2_{e\gamma}$ is allowed to vary. Note that we do not model effect heterogeneity across states: such heterogeneity would likely make aggregating effect estimates inappropriate (see Section 6).

We consider two treated states (i.e., two cohorts) under a range of scenarios, varying the number of control states, T_{pre}, T_{post}, Δ, within- and between-person correlations, and the percent of individuals in each control state shared between cohorts (assumed constant). For simplicity, we set $\rho_\gamma = \rho$, $\phi_\gamma = \phi$ and $\psi_\gamma = \psi$ for all states γ in each simulation setting. We only consider settings in which the generated correlation structure matches Equation (7). The correlation estimate and variance correction developed here explicitly assume that structure and do not apply to other, non-exchangeable structures. When the covariance structure is correctly specified, we expect an unbiased estimate of the correlation between cohort-specific DiD estimates. We also expect to see nominal coverage of confidence intervals for the inverse-variance weighted $\widehat{\text{ATT}}_{\text{ivw}}$ when we use the correlation-corrected variance formula in Equation (16), with deviations from nominal coverage increasing as magnitude of the between-cohort correlation becomes larger. Reported correlations are averages of 100 estimates, each of which is computed from 100 pairs of simulated $\widehat{ATT}$s.

Our hypotheses are confirmed in Table 1, which presents results for settings with 3 control states (see Appendix B for 10 controls). Over a variety of scenarios, we see very small bias in the estimated correlation. Of primary interest, however, are the corrected standard errors of the inverse variance weighted average ATT over both simulated treated states. Both $\widehat{\text{ATT}}_{\text{ivw}}$ and $\widehat{\text{ATT}}_{\text{gls}}$ are unbiased, but have different standard errors, and therefore coverage (Figure 4). As the magnitude of the correlation increases, uncorrected (IVW-based) coverage suffers. When correlations are negative, the uncorrected standard errors of $\widehat{\text{ATT}}_{\text{ivw}}$ are too large; when positive, too small. The correlation-corrected intervals around $\widehat{\text{ATT}}_{\text{gls}}$, though, achieve nominal coverage across scenarios.

We turn now to the medical cannabis laws study. In order to estimate the between-ATT

							$\text{Cor}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right)$		$\widehat{\text{ATT}}_{\text{ivw}}$			$\widehat{\text{ATT}}_{\text{gls}}$		
T_{pre}	T_{post}	Δ	% Shared	ρ	ϕ	ψ	True Cor.	Est. Bias	Bias	SE	95% Covg.	Bias	SE	95% Covg.
1	1	1	0.25	0.10	0.06	0.02	-0.108	-0.00	-0.00	0.27	0.962	-0.00	0.26	0.950
				0.60	0.40	0.20	-0.124	0.00	0.01	1.16	0.961	0.01	1.09	0.944
			0.75	0.10	0.06	0.02	-0.119	0.01	0.00	0.27	0.965	0.00	0.26	0.950
				0.60	0.40	0.20	-0.125	0.01	-0.01	1.16	0.965	-0.01	1.09	0.950
		2	0.25	0.10	0.06	0.02	0.000	0.01	-0.00	0.27	0.952	-0.00	0.27	0.952
				0.60	0.40	0.20	0.000	-0.01	0.00	1.16	0.953	0.00	1.16	0.953
			0.75	0.10	0.06	0.02	0.000	-0.00	-0.00	0.27	0.951	-0.00	0.27	0.951
				0.60	0.40	0.20	0.000	0.01	-0.01	1.16	0.949	-0.01	1.16	0.949
5	5	3	0.25	0.10	0.06	0.02	0.022	0.00	0.00	0.12	0.944	0.00	0.12	0.947
				0.60	0.40	0.20	0.025	-0.01	-0.00	0.52	0.948	-0.00	0.53	0.951
			0.75	0.10	0.06	0.02	0.024	0.01	-0.00	0.12	0.946	-0.00	0.12	0.950
				0.60	0.40	0.20	0.025	-0.00	-0.00	0.52	0.949	-0.00	0.53	0.952
		6	0.25	0.10	0.06	0.02	-0.087	0.01	0.00	0.12	0.958	0.00	0.12	0.951
				0.60	0.40	0.20	-0.099	0.01	0.00	0.52	0.958	0.00	0.49	0.943
			0.75	0.10	0.06	0.02	-0.096	-0.00	-0.00	0.12	0.960	-0.00	0.12	0.951
				0.60	0.40	0.20	-0.100	0.00	0.00	0.52	0.961	0.00	0.49	0.949
7	3	3	0.25	0.10	0.06	0.02	-0.028	0.00	-0.00	0.13	0.955	-0.00	0.13	0.951
				0.60	0.40	0.20	-0.032	0.02	0.00	0.57	0.953	0.00	0.56	0.948
			0.75	0.10	0.06	0.02	-0.031	-0.01	-0.00	0.13	0.957	-0.00	0.13	0.955
				0.60	0.40	0.20	-0.032	-0.01	-0.00	0.57	0.955	-0.00	0.56	0.950
		6	0.25	0.10	0.06	0.02	-0.056	0.02	0.00	0.13	0.955	0.00	0.13	0.948
				0.60	0.40	0.20	-0.064	0.00	-0.00	0.57	0.956	-0.00	0.55	0.948
			0.75	0.10	0.06	0.02	-0.061	0.00	-0.00	0.13	0.955	-0.00	0.13	0.948
				0.60	0.40	0.20	-0.064	-0.00	0.01	0.57	0.959	0.01	0.55	0.952

Table 1: Simulated between-estimate correlations along with standard error and 95% confidence interval coverage for aggregated estimates of $\widehat{\text{ATT}}$ for a variety of generative model parameters, 100 individuals per state, and 3 control states. Reported correlations are averages of 100 estimates generated from 100 simulations.

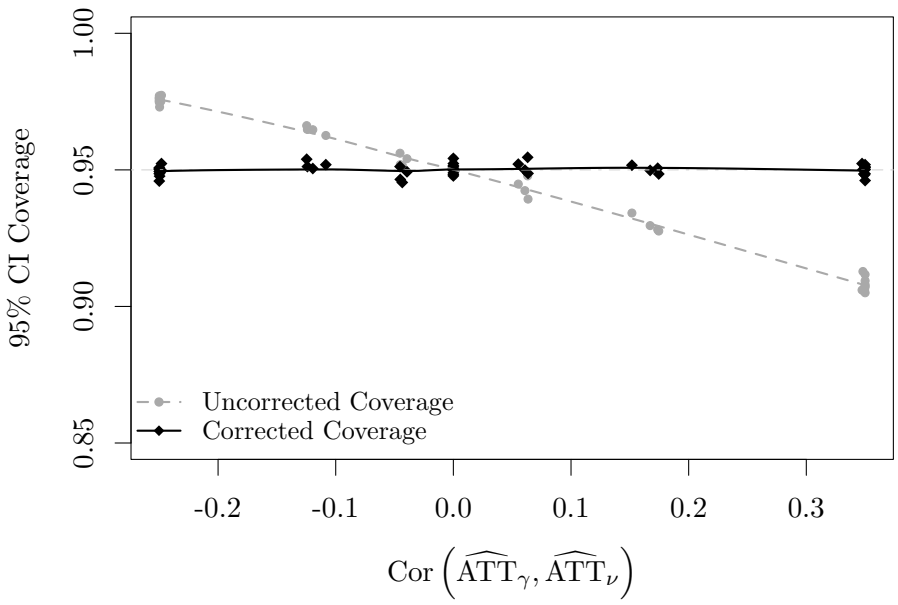

Figure 4: 95% confidence interval coverage when applying and not applying the correlation correction as a function of between-estimate correlation. Coverage is estimated based on 10,000 simulations of cohorts with 10 total measurements, 5 pre-treatment and 5 post-treatment, with 1 control unit. Lines are loess smoothers with bandwidth 3/4.

correlations due to shared control individuals, we need estimates of the within- and between-person correlations ρ_γ, ϕ_γ and ψ_γ for each state $\gamma \in \Xi$. While the formulas in Equations (8) and (10) allow for state-specific correlation estimates, obtaining such estimates can be challenging, particularly in larger datasets: the medical cannabis laws study's analytic sample consists of 583,820 individuals measured for at least 84 months (treated and disjoint control individuals contribute exactly 84; shared control individuals contribute strictly more than 84). As such, we follow the literature for cluster-randomized trials (Ouyang, Hemming, Li & Taljaard 2023, Kasza et al. 2019), and estimate these correlations averaged over all states using a mixed-effects modeling approach, specifically by fitting the model

$$Y_{\gamma it} = \beta_0 + \beta_{1,t} + \beta_2 A_{\gamma t} + b_i + b_\gamma + b_{\gamma t} + \epsilon_{\gamma it}, \tag{18}$$

where $A_{\gamma t}$ is a treatment indicator in Equation (3), b_i is a person-specific random effect, b_γ a state-specific random effect, and $b_{\gamma t}$ a state-time-specific random effect as in Equation (17). Correlation estimates are functions of estimates of the variances of the random effects; see Table 1 of Ouyang, Hemming, Li & Taljaard (2023) for details.

For this reanalysis, we focus on three outcomes: the proportion of chronic non-cancer pain patients receiving any opioid prescription in a given month; the proportion receiving any non-cannabis, non-opioid prescription analgesic in a given month; and the proportion receiving any procedure for chronic pain in a given month. These are state-time outcomes aggregated up from

	$\hat{\rho}$	$\hat{\phi}$	$\hat{\psi}$
Any Opioid Rx	0.463	0.024	0.023
Any Non-Opioid Rx	0.318	0.014	0.013
Any Procedure	0.181	0.006	0.004

Table 2: Estimated within- and between-person correlations in the medical cannabis laws study.

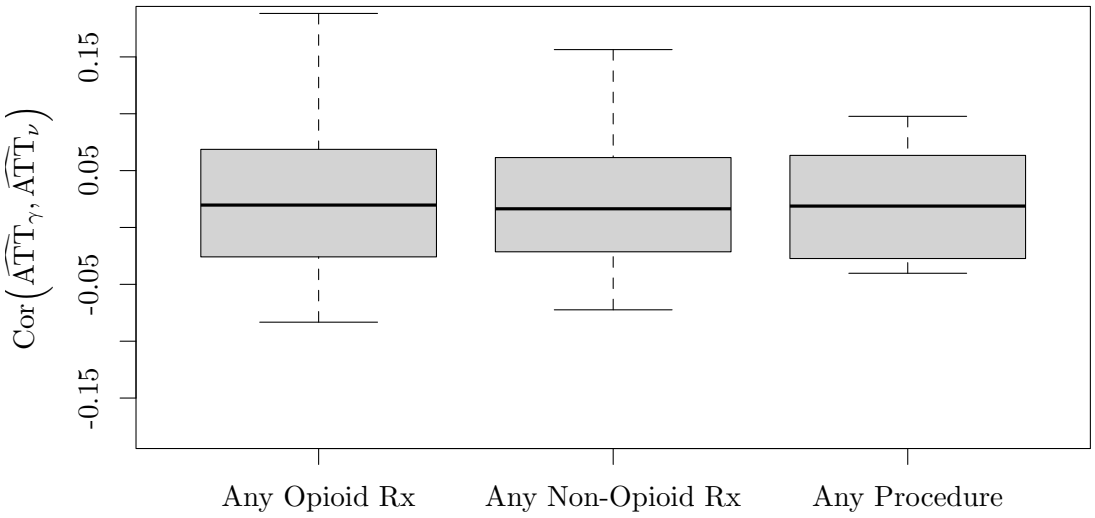

Figure 5: Boxplots of computed correlations between state-specific ATT estimates for each outcome in the medical cannabis laws study; $(\gamma, \nu) \in \Xi_{\text{tx}} \times \Xi_{\text{tx}}$ where $\Xi_{\text{tx}} = \{\text{CT}, \ldots, \text{LA}\}$.

individual-level data, and are reported in percentage points. The block exchangeable correlation structure in Equation (7) is scientifically reasonable here, as we expect within- and between-person relationships in the outcomes of interest to be relatively stable over time in the population of interest (commercially-insured U.S. adults with a chronic non-cancer pain diagnosis). Table 2 contains estimates of ρ, ϕ, and ψ on average across all states for each outcome, averaged across 100 resamples of 0.1% of the 583,820 individuals in the full sample such that each resample is balanced in the number of individuals per state. Note that our assumption that $\rho > \phi > \psi$ appears valid based on these estimates. In Appendix C we present per-cohort sample sizes as well as counts of disjoint and shared control individuals per cohort pair.

Using the estimates in Table 2, we can compute pairwise correlations between state-specific $\widehat{\text{ATT}}$s, which are summarized for each outcome in Figure 5. We note that the correlations are not centered at zero. In the medical cannabis laws study, $T_{\text{pre}} = 48$ and $T_{\text{post}} = 36$, correlations are positive for $\Delta < \Delta^* = 27.2$ months, and the median spacing between cohorts is 21.5 months. Also note that some of the correlations are quite sizeable: values for the outcome indicating receipt of any opioid prescription in a given month range from -0.095 to 0.185.

In our reanalysis, updated point estimates are not meaningfully different from those reported

by McGinty et al. (2023), but the standard errors that account for correlation induced by control individuals shared over the 12 stacked DiD analyses are larger (and thus confidence intervals are wider); inference remains the same. After accounting for correlation due to shared control individuals, we estimate an average difference of 0.04 percentage points (95% CI -0.14 to 0.22 percentage points), 0.04 percentage points (CI -0.16 to 0.24 percentage points), and -0.17 percentage points (CI -0.44, 0.11 percentage points) in the proportion of individuals receiving any opioid prescription, any nonopioid prescription, and any chronic pain proceudre, respectively, attributable to state medical cannabis laws in a given month during the first three years of law implementation. These effects are small, and the confidence intervals remain narrow after accounting for between-estimate correlation, ruling out meaningful effects of medical cannabis laws on chronic non-cancer pain treatment in either direction. These results are both quantitatively and qualitatively similar to the unadjusted results in McGinty et al. (2023); our correlation-corrected confidence intervals are 0.03, 0.04, and 0.05 percentage points (about 10%) wider than the uncorrected intervals (see Figure 6, which recreates Figure 1 of McGinty et al. (2023)). An important note is that the original analysis uses the augmented synthetic control method to obtain state-specific $\widehat{\text{ATT}}_\gamma$s: this approach is conceptually similar to DiD with weights on the control units (Ben-Michael, Feller & Rothstein 2021). While our current results do not accommodate such weights, we do not believe the substantive findings would not meaningfully change by doing so.

6 Discussion

We have developed a method for correcting standard errors for average ATTs in a stacked DiD study in the presence of shared control individuals and when individual-level data is available. The reuse of data from individuals in control states across multiple stacked analyses induces meaningful correlation between estimates that can be quantified and accounted for when pooling effect estimates. Assuming a block-exchangeable within- and between-person correlation structure, the sign of the correlation between two DiD effect estimates is entirely determined by timing – the duration of the units' study periods and how much or little overlap there is in those study periods in calendar time. Failure to account for this correlation can lead to over- or under-estimation of the standard error of the pooled ATT estimate, depending on the sign of the correlation.

We note that this method applies only in scenarios in which scientific interest is in a pooled ATT estimate averaged across multiple treated units. In the medical cannabis laws example, we found no evidence of effect heterogeneity across policy-implementing states, and therefore comfortably pooled those results. In the presence of meaningful differences in effects, scientists may not want to focus on an average ATT, and instead report unit-specific $\widehat{\text{ATT}}_\gamma$s alongside deeper qualitative analysis of the results.

Though we found no substantive difference in corrected and uncorrected standard errors in the medical cannabis laws study, we believe the correction could be quite impactful in other settings in which states implement policies much closer together or farther apart in time (i.e., when Δs are

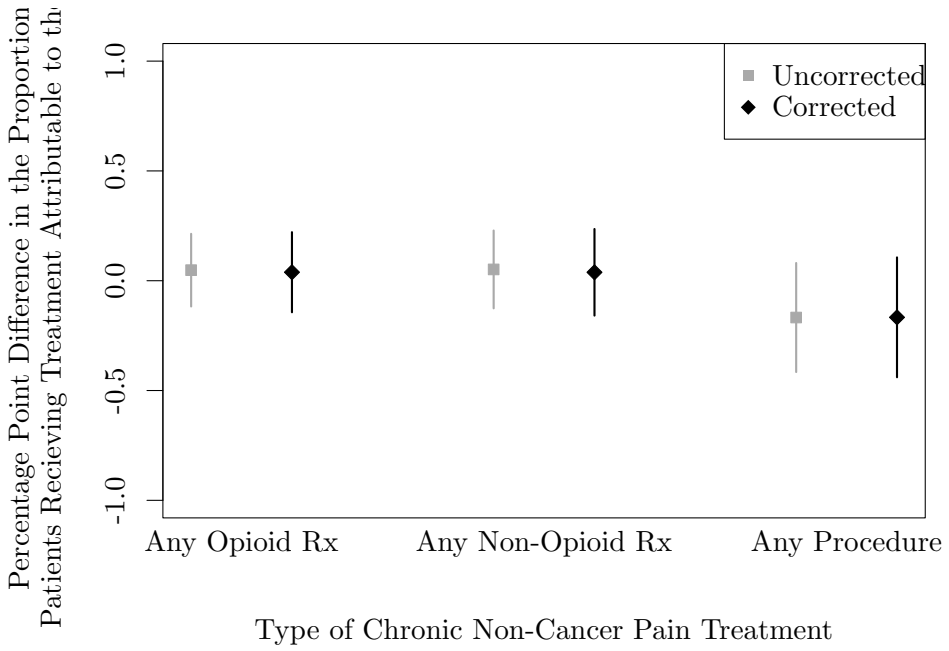

Figure 6: Percentage point differences in proportions of patients receiving chronic non-cancer pain treatment attributable to medical cannabis laws on average over the first 3 years of law implementation, with and without the correction for correlation due to shared controls. Any opioid Rx: McGinty et al. reported an estimated effect of 0.05 percentage points (95% CI, -0.12 to 0.21 percentage points); our correlation-corrected estimate is 0.04 percentage points (95% CI, -0.14 to 0.22 percentage points). Any non-opioid Rx: McGinty et al., 0.05 percentage points (95% CI, -0.13 to 0.23 percentage points); corrected, 0.04 percentage points (95% CI -0.16 to 0.24 percentage points). Any procedure: McGinty et al., -0.17 percentage points (95% CI -0.42 to 0.08 percentage points); corrected, -0.17 percentage points (95% CI -0.44, 0.11 percentage points).

consistently small or large). As an example, if we pooled over just CT, MN, and NY, the width of the corrected confidence interval for the proportion of patients receiving any opioid prescription would more than triple compared to the uncorrected interval (0.33 percentage points versus 1.06 percentage points). In a setting with a true policy effect, this may be enough to change inference. Additionally, we may see more meaningful changes in the medical cannabis law confidence intervals under a different correlation structure.

An important limitation is the imposition of a particular correlation structure, though its choice was motivated by similar work in the cluster-randomized trials literature (Kasza et al. 2019, Ouyang, Kulkarni, Protopopoff, Li & Taljaard 2023, Ouyang, Hemming, Li & Taljaard 2023). The formulae presented above do not, for instance, allow for decaying within- and between-person correlations over time. An alternative approach might be the bootstrap; however, it is unclear how one should perform resampling in this setting to preserve the data structure involving shared controls. Bootstrapping the entire dataset for all states simultaneously was computationally infeasible for the medical cannabis laws study; this difficulty will likely translate to most policy evaluations with individual-level data. Pairwise resampling for each combination of cohorts would be more computationally feasible and adapting a cluster bootstrap to this setting may be promising, though future work is needed. We therefore trade an assumption on correlation structure for interpretability — in that we developed a closed form of the between-estimate correlation — and speed.

Future work will accommodate additional correlation structures, including those with autoregressive decay over time, and incorporate synthetic control weights to more accurately reflect realistic correlation structures in data used for health policy evaluation. Additionally, software to compute the between-estimate correlation is available at https://github.com/nickseewald/didsharedctrls.

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A Derivation of DiD Correlation Formula

We continue with the notation established in the main text. Consider two treated states $\gamma, \nu \in \Xi_{\text{tx}}$, each of which have study periods of length $T = T_{\text{pre}} + T_{\text{post}}$ which begin $\Delta = |t_{*\gamma} - t_{*\nu}|$ periods apart. Recall the plug-in DiD estimator for treated state γ:

$$\widehat{\text{ATT}}_\gamma = \left(\bar{Y}_{\gamma,\{t\geq t_{*\gamma}\}} - \bar{Y}_{\gamma,\{t<t_{*\gamma}\}}\right) - \left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}} - \bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}\right), \tag{19}$$

where, for example,

$$\bar{Y}_{\text{ctrl},\{t\geq t_*\}} = \frac{1}{N_\gamma^{\text{ctrl}} T_{\text{post}}} \sum_{\zeta\in\Xi_{\text{ctrl}}} \sum_{t=t_{*\gamma}}^{t_{T\gamma}} \sum_{i=1}^{N_\gamma(\zeta)} Y_{\zeta it} \tag{20}$$

is the mean outcome over all control individuals in cohort $\mathcal{C}_\gamma$ at all post-treatment measurement occasions.

For simplicity, we assume a block exchangeable correlation structure for all states. For two individuals i, j in the same state $\gamma \in \Xi$ and timepoints t and s, $\text{Cor}(Y_{\gamma it}, Y_{\gamma is}) = \rho_\gamma$ (within-person correlation), $\text{Cor}(Y_{\gamma it}, Y_{\gamma jt}) = \phi_\gamma$ (within-period correlation), and $\text{Cor}(Y_{\gamma it}, Y_{\gamma js}) = \psi_\gamma$ (between-period correlation). For a state γ, then, the correlation matrix for all observations is block-diagonal with $\text{Exch}_T(\rho_\gamma)$ correlation matrices on the diagonal and off-diagonal blocks $\psi_\gamma \mathbf{1}_T \mathbf{1}_T^\top + (\phi_\gamma - \psi_x i) I_T$, where $\text{Exch}_T(\rho)$ is a $T \times T$ matrix with 1's on the diagonal and all off-diagonal elements are ρ, $\mathbf{1}_T$ is a T-vector of 1's, and I_T is the $T \times T$ identity matrix. This is depicted visually in equation (7).

A.1 Derivation of DiD Variance

We start by deriving an expression for the variance of the DiD estimator of ATT_γ in equation (19). We have

$$\begin{aligned}\text{Var}\left(\widehat{\text{ATT}}_\gamma\right) = \text{Var}\left(\bar{Y}_{\gamma,\{t\geq t_{*\gamma}\}}\right) + \text{Var}\left(\bar{Y}_{\gamma,\{t<t_{*\gamma}\}}\right) + \text{Var}\left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}}\right) + \text{Var}\left(\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}\right) \\ - 2\text{Cov}\left(\bar{Y}_{\gamma,\{t\geq t_{*\gamma}\}}, \bar{Y}_{\gamma,\{t<t_{*\gamma}\}}\right) - 2\text{Cov}\left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}}, \bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}\right).\end{aligned} \tag{21}$$

Note that there are no covarances between γ and control states since, under the assumption that individuals living in different states are independent of one another, they are identically zero.

For treated state γ,

$$\begin{aligned}\text{Var}\left(\bar{Y}_{\gamma,\{t\geq t_{*\gamma}\}}\right) &= \text{Var}\left(\frac{1}{N_\gamma(\gamma) T_{\text{post}}} \sum_{i=1}^{N_\gamma(\gamma)} \sum_{t=t_{*\gamma}}^{t_{T\gamma}} Y_{\gamma it}\right) \\ &= \left(N_\gamma(\gamma) T_{\text{post}}\right)^{-2} \left(\sum_i \sum_{t\geq t_{*\gamma}} \text{Var}(Y_{\gamma it}) + \sum_i \sum_{t\neq t'\geq t_{*\gamma}} \text{Cov}\left(Y_{\gamma it}, Y_{\gamma it'}\right)\right. \\ &\qquad \left. + \sum_{i\neq j} \sum_{t\geq t_{*\gamma}} \text{Cov}(Y_{\gamma it}, Y_{\gamma jt}) + \sum_{i\neq j} \sum_{t\neq t'\geq t_{*\gamma}} \text{Cov}\left(Y_{\gamma it}, Y_{\gamma jt'}\right)\right) \\ &= \left(N_\gamma(\gamma) T_{\text{post}}\right)^{-2} \left(N_\gamma(\gamma) T_{\text{post}} + N_\gamma(\gamma) T_{\text{post}}(T_{\text{post}} - 1)\rho_\gamma + T_{\text{post}} N_\gamma(\gamma) \left(N_\gamma(\gamma) - 1\right) \phi_\gamma + \right. \\ &\qquad \left. + T_{\text{post}}(T_{\text{post}} - 1) N_\gamma(\gamma) \left(N_\gamma(\gamma) - 1\right) \psi_\gamma\right) \sigma_\gamma^2 \\ &= \left(N_\gamma(\gamma) T_{\text{post}}\right)^{-1} \left(1 + (T_{\text{post}} - 1)\rho_\gamma + \left(N_\gamma(\gamma) - 1\right) \left(\phi_\gamma + (T_{\text{post}} - 1)\psi_\gamma\right)\right) \sigma_\gamma^2.\end{aligned} \tag{22}$$

Note that $\text{Var}\left(\bar{Y}_{\gamma,\{t<t_{*\gamma}\}}\right)$ is identical to the expression in equation (22), replacing T_{post} with T_{pre}.

Turning now to the control states, we have

$$\mathrm{Var}\big(\bar{Y}_{\mathrm{ctrl},\{t\geq t_{*\gamma}\}}\big) = \mathrm{Var}\left(\frac{1}{N_\gamma^{\mathrm{ctrl}} T_{\mathrm{post}}} \sum_{\zeta\in\Xi_{\mathrm{ctrl}}} \sum_{t=t_{*\gamma}}^{t_{T\gamma}} \sum_{i=1}^{N_\gamma(\zeta)} Y_{\zeta it}\right) \tag{23}$$

$$= \Big(N_\gamma^{\mathrm{ctrl}} T_{\mathrm{post}}\Big)^{-2} \sum_{\zeta\in\Xi_{\mathrm{ctrl}}} \left(\sum_i \sum_{t\geq t_{*\gamma}} \mathrm{Var}(Y_{\gamma it}) + \sum_i \sum_{t\neq t'\geq t_{*\gamma}} \mathrm{Cov}\big(Y_{\gamma it}, Y_{\gamma it'}\big) \right.$$
$$\left. + \sum_{i\neq j} \sum_{t\geq t_{*\gamma}} \mathrm{Cov}(Y_{\gamma it}, Y_{\gamma jt}) + \sum_{i\neq j} \sum_{t\neq t'\geq t_{*\gamma}} \mathrm{Cov}\big(Y_{\gamma it}, Y_{\gamma jt'}\big) \right) \tag{24}$$

$$= \Big(N_\gamma^{\mathrm{ctrl}} T_{\mathrm{post}}\Big)^{-2} \sum_{\zeta\in\Xi_{\mathrm{ctrl}}} \left(N_\gamma(\zeta) T_{\mathrm{post}} + N_\gamma(\zeta) T_{\mathrm{post}}(T_{\mathrm{post}}-1)\rho_\zeta + N_\gamma(\zeta)\left(N_\gamma(\zeta)-1\right) T_{\mathrm{post}}\phi_\zeta \right.$$
$$\left. + N_\gamma(\zeta)\left(N_\gamma(\zeta)-1\right) T_{\mathrm{post}}(T_{\mathrm{post}}-1)\psi_\zeta \right) \sigma_\zeta^2$$

$$= \Big(N_\gamma^{\mathrm{ctrl}} T_{\mathrm{post}}\Big)^{-2} \sum_{\zeta\in\Xi_{\mathrm{ctrl}}} N_\gamma(\zeta) T_{\mathrm{post}} \left(1 + \left(T_{\mathrm{post}}-1\right)\rho_\zeta + \left(N_\gamma(\zeta)-1\right)\left(\phi_\zeta + \left(T_{\mathrm{post}}-1\right)\psi_\zeta\right)\right)\sigma_\zeta^2. \tag{25}$$

Note that equation (24) follows from equation (23) under the assumption that individuals living in different states are independent of one another. As with the treated states, the pre-treatment analogue of equation (25) is identical, replacing T_{post} with T_{pre}.

We now derive expressions for the covariance terms in equation (21) under the covariance structure on the data given in equation (7). Note that for simplicity we use $(t_{*\gamma}-1)$ as shorthand to refer to the measurement occasion just prior to treatment initiation / policy implementation; we do not require that measurements be one time unit apart. We have

$$\mathrm{Cov}\big(\bar{Y}_{\gamma,\{t\geq t_{*\gamma}\}}, \bar{Y}_{\gamma,\{t<t_{*\gamma}\}}\big) = \mathrm{Cov}\left(\frac{1}{N_\gamma(\gamma) T_{\mathrm{post}}} \sum_{t=t_{*\gamma}}^{t_{T\gamma}} \sum_{i=1}^{N_\gamma(\gamma)} Y_{\gamma it}, \frac{1}{N_\gamma(\gamma) T_{\mathrm{pre}}} \sum_{t=t_{1\gamma}}^{t_{*\gamma}-1} \sum_{i=1}^{N_\gamma(\gamma)} Y_{\gamma it}\right)$$
$$= \frac{1}{N_\gamma(\gamma)^2 T_{\mathrm{pre}} T_{\mathrm{post}}} \sum_{t=t_{1\gamma}}^{t_{*\gamma}-1} \sum_{t'=t_{*\gamma}}^{t_{T\gamma}} \left(\sum_{i=1}^{N_\gamma(\gamma)} \mathrm{Cov}\big(Y_{\gamma it}, Y_{\gamma it'}\big) + \sum_{i\neq j} \mathrm{Cov}\big(Y_{\gamma it}, Y_{\gamma jt'}\big) \right)$$
$$= \frac{1}{N_\gamma(\gamma)^2 T_{\mathrm{pre}} T_{\mathrm{post}}} \sum_{t=t_{1\gamma}}^{t_{*\gamma}-1} \sum_{t'=t_{*\gamma}}^{t_{T\gamma}} \left(N_\gamma(\gamma)\rho_\gamma + N_\gamma(\gamma)\left(N_\gamma(\gamma)-1\right)\psi_\gamma\right)\sigma_\gamma^2$$
$$= \frac{1}{N_\gamma(\gamma)} \left(\rho_\gamma + \left(N_\gamma(\gamma)-1\right)\psi_\gamma\right)\sigma_\gamma^2. \tag{26}$$

Similarly, for the control states, we have

$$\operatorname{Cov}\left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}},\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}\right)=\operatorname{Cov}\left(\frac{1}{N_{\gamma}^{\text{ctrl}}T_{\text{post}}}\sum_{\zeta\in\Xi_{\text{ctrl}}}\sum_{t=t_{*\gamma}}^{t_{T\gamma}}\sum_{i=1}^{N_{\gamma}(\zeta)}Y_{\zeta it},\frac{1}{N_{\gamma}^{\text{ctrl}}T_{\text{post}}}\sum_{\zeta\in\Xi_{\text{ctrl}}}\sum_{t=t_{1\gamma}}^{t_{*\gamma}-1}\sum_{i=1}^{N_{\gamma}(\zeta)}Y_{\zeta it}\right) \tag{27}$$

$$=\frac{1}{\left(N_{\gamma}^{\text{ctrl}}\right)^{2}T_{\text{pre}}T_{\text{post}}}\sum_{\zeta\in\Xi_{\text{ctrl}}}\sum_{t=t_{1\gamma}}^{t_{*\gamma}-1}\sum_{t'=t_{*\gamma}}^{t_{T\gamma}}\left(\sum_{i=1}^{N_{\gamma}(\zeta)}\operatorname{Cov}\left(Y_{\zeta it},Y_{\zeta it'}\right)+\sum_{i\neq j}\operatorname{Cov}\left(Y_{\zeta it},Y_{\zeta jt'}\right)\right) \tag{28}$$

$$=\frac{1}{\left(N_{\gamma}^{\text{ctrl}}\right)^{2}T_{\text{pre}}T_{\text{post}}}\sum_{\zeta\in\Xi_{\text{ctrl}}}\sum_{t=t_{1\gamma}}^{t_{*\gamma}-1}\sum_{t'=t_{*\gamma}}^{t_{T\gamma}}\left(N_{\gamma}(\zeta)\rho_{\zeta}+N_{\gamma}(\zeta)\left(N_{\gamma}(\zeta)-1\right)\psi_{\zeta}\right)\sigma_{\zeta}^{2}$$

$$=\frac{1}{\left(N_{\gamma}^{\text{ctrl}}\right)^{2}}\sum_{\zeta\in\Xi_{\text{ctrl}}}\left(N_{\gamma}(\zeta)\rho_{\zeta}+N_{\gamma}(\zeta)\left(N_{\gamma}(\zeta)-1\right)\psi_{\zeta}\right)\sigma_{\zeta}^{2}, \tag{29}$$

with equation (28) following from equation (27) under independence of states.

Now, using equations (22) and (26) (and the pre-treatment analogue of equation (22)), we have

$$\begin{aligned}
&\operatorname{Var}\left(\bar{Y}_{\gamma,\{t\geq t_{*\gamma}\}}\right)+\operatorname{Var}\left(\bar{Y}_{\gamma,\{t<t_{*\gamma}\}}\right)-2\operatorname{Cov}\left(\bar{Y}_{\gamma,\{t\geq t_{*\gamma}\}},\bar{Y}_{\gamma,\{t<t_{*\gamma}\}}\right)\\
&=\frac{\sigma_{\gamma}^{2}}{\left(N_{\gamma}(\gamma)T_{\text{pre}}T_{\text{post}}\right)^{2}}\left[N_{\gamma}(\gamma)\left(T_{\text{post}}^{2}T_{\text{pre}}+T_{\text{post}}^{2}T_{\text{pre}}^{2}\rho_{\gamma}-T_{\text{post}}^{2}T_{\text{pre}}\rho_{\gamma}+T_{\text{post}}T_{\text{pre}}^{2}\right.\right.\\
&\qquad\left.+T_{\text{post}}^{2}T_{\text{pre}}^{2}\rho_{\gamma}-T_{\text{post}}T_{\text{pre}}^{2}\rho_{\gamma}-2T_{\text{post}}^{2}T_{\text{pre}}^{2}\rho_{\gamma}\right)\\
&\qquad N_{\gamma}(\gamma)\left(N_{\gamma}(\gamma)-1\right)\left(T_{\text{post}}^{2}T_{\text{pre}}\phi_{\gamma}+T_{\text{post}}T_{\text{pre}}^{2}\phi_{\gamma}+T_{\text{post}}^{2}T_{\text{pre}}^{2}\psi_{\gamma}-T_{\text{post}}^{2}T_{\text{pre}}\psi_{\gamma}\right.\\
&\qquad\left.\left.+T_{\text{post}}^{2}T_{\text{pre}}^{2}\psi_{\gamma}-T_{\text{post}}T_{\text{pre}}^{2}\psi_{\gamma}-2T_{\text{pre}}^{2}T_{\text{post}}^{2}\psi_{\gamma}\right)\right]\\
&=\frac{\sigma_{\gamma}^{2}}{\left(N_{\gamma}(\gamma)T_{\text{pre}}T_{\text{post}}\right)^{2}}\left(N_{\gamma}(\gamma)T_{\text{post}}T_{\text{pre}}\left(T_{\text{pre}}+T_{\text{post}}\right)\right)\left(\left(1-\rho\right)+\left(N_{\gamma}(\gamma)-1\right)\left(\phi_{\gamma}-\psi_{\gamma}\right)\right)\\
&=\frac{T_{\text{pre}}+T_{\text{post}}}{\left(N_{\gamma}(\gamma)\right)^{2}T_{\text{pre}}T_{\text{post}}}\left(N_{\gamma}(\gamma)\left(1-\rho\right)+N_{\gamma}(\gamma)\left(N_{\gamma}(\gamma)-1\right)\left(\phi_{\gamma}-\psi_{\gamma}\right)\right)\sigma_{\gamma}^{2}.
\end{aligned} \tag{30}$$

Proceeding similarly with equations (25) and (29) (and the pre-treatment analogue of equation (25)), we have

$$\begin{aligned}
&\operatorname{Var}\left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}}\right)+\operatorname{Var}\left(\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}\right)-2\operatorname{Cov}\left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}},\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}\right)\\
&\qquad=\frac{T_{\text{pre}}+T_{\text{post}}}{\left(N_{\gamma}^{\text{ctrl}}\right)^{2}T_{\text{pre}}T_{\text{post}}}\sum_{\zeta\in\Xi_{\text{ctrl}}}\left(N_{\gamma}(\zeta)\left(1-\rho_{\zeta}\right)+N_{\gamma}(\zeta)\left(N_{\gamma}(\zeta)-1\right)\left(\phi_{\zeta}-\psi_{\zeta}\right)\right)\sigma_{\zeta}^{2}.
\end{aligned} \tag{31}$$

Finally, summing equations (30) and (31), we have

$$\boxed{\operatorname{Var}\left(\widehat{\text{ATT}}_{\gamma}\right)=\frac{T_{\text{pre}}+T_{\text{post}}}{T_{\text{pre}}T_{\text{post}}}\sum_{\zeta\in(\{\gamma\}\cup\Xi_{\text{ctrl}})}\frac{\sigma_{\zeta}^{2}}{\left(N_{\gamma}^{A_{\gamma}(\zeta)}\right)^{2}}\left[N_{\gamma}(\zeta)\left(1-\rho_{\zeta}\right)+N_{\gamma}(\zeta)\left(N_{\gamma}(\zeta)-1\right)\left(\phi_{\zeta}-\psi_{\zeta}\right)\right],} \tag{32}$$

where $A_{\zeta}=\mathbb{1}\{\zeta\in\Xi_{\text{tx}}\}$ is an indicator for whether state ζ was ever treated such that $N_{\gamma}^{A_{\zeta}}=A_{\zeta}N_{\gamma}(\gamma)+(1-A_{\zeta})N_{\gamma}^{\text{ctrl}}$. That is, if $\zeta\in\Xi_{\text{ctrl}}$, then $N_{\gamma}^{A_{\zeta}}=N_{\gamma}^{\text{ctrl}}$ is the total number of individuals in control states that contribute to $\widehat{ATT}_{\gamma}$. Similarly, if $\zeta\in\Xi_{\text{tx}}$ (i.e., $\zeta=\gamma$), then $N_{\gamma}^{A_{\zeta}}=N_{\gamma}^{\text{tx}}=N_{\gamma}(\gamma)$.

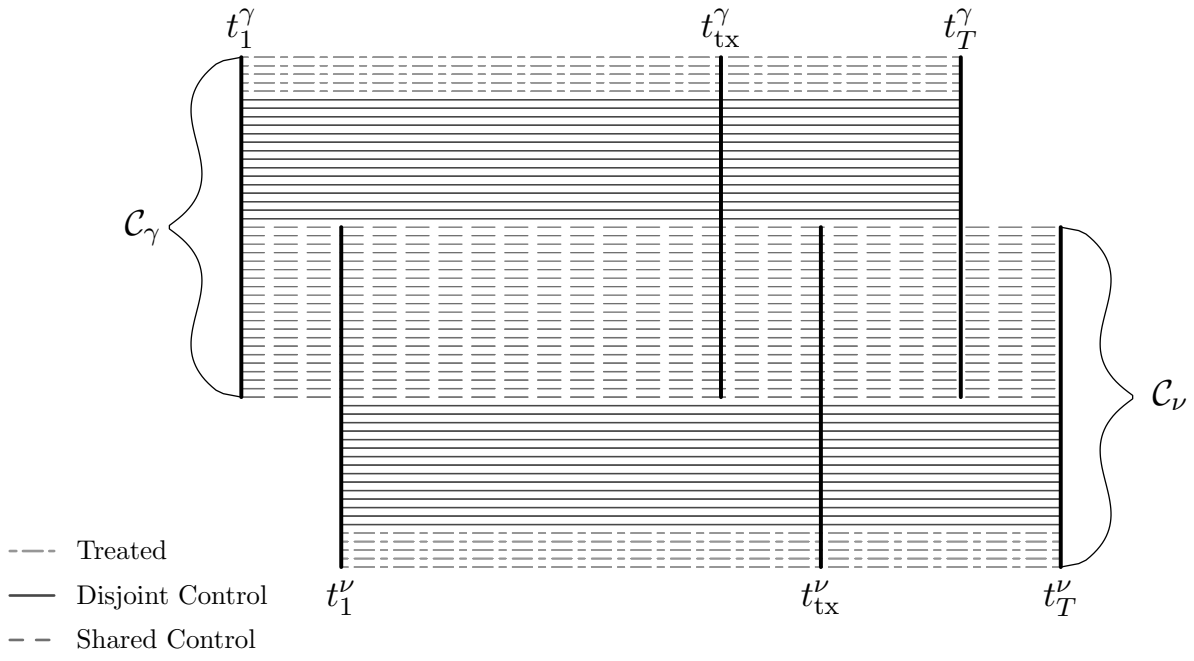

Figure 7: A schematic depiction of two cohorts' overlapping study periods and shared control individuals. Horizontal lines represent individual "timelines" over which data are collected from that individual. Dot-dashed lines represent individuals in a treated state, solid lines represent individuals in a control state who contribute to exactly one of $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$ ("disjoint" control individuals), and dashed lines shared control individuals.

A.2 Derivation of Covariance between DiD Estimates

Consider two cohorts, $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$, separated by Δ measurement occasions. By linearity of covariance and independence of individuals from different states,

$$\begin{aligned}\text{Cov}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right) = \text{Cov}\left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}}, \bar{Y}_{\text{ctrl},\{t\geq t_{*\nu}\}}\right) + \text{Cov}\left(\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}, \bar{Y}_{\text{ctrl},\{t<t_{*\nu}\}}\right) \\ - \text{Cov}\left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}}, \bar{Y}_{\text{ctrl},\{t<t_{*\nu}\}}\right) - \text{Cov}\left(\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}, \bar{Y}_{\text{ctrl},\{t\geq t_{*\nu}\}}\right). \quad (33)\end{aligned}$$

Notice that the covariance only involves data from control individuals: since individuals in treated states γ and ν are independent of individuals in every other state, terms involving $\bar{Y}_{\gamma,\{t\}}$ and $\bar{Y}_{\nu,\{t\}}$ drop out of the covariance.

Consider figure 7, which depicts overlap between cohorts $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$ when $\Delta < T_{\text{post}} < T_{\text{pre}}$. In the figure, dot-dashed lines represent study timelines for treated individuals in either state γ or state ν. Solid lines represent timelines for "disjoint" control individuals in some control state $\zeta \in \Xi_{\text{ctrl}}$ who contribute to *either* the cohort for state γ or for state ν but not both (i.e., individuals $i \in (\mathcal{I}_\gamma / \mathcal{I}_\gamma(\gamma)) \sqcup (\mathcal{I}_\nu / \mathcal{I}_\nu(\nu))$), and dashed lines represent "shared control" individuals that contribute to cohorts for both γ and ν (i.e., individuals $i \in (\mathcal{I}_\gamma \cap \mathcal{I}_\nu)$. Similar diagrams can be created for other configurations of T_{pre}, T_{post}, and Δ.

We begin by dividing the person-time contributed by control individuals to each cohort into a number of disjoint "windows". Definitions of the windows are given in table 3. Data from disjoint control individuals fall into one of 4 possible windows: pre- or post-treatment time in either $\mathcal{C}_\gamma$ or $\mathcal{C}_\nu$. As an example, we denote the set of *disjoint* control person-time in $\mathcal{C}_\gamma$'s pre-treatment period as $\mathcal{D}_{\text{pre}}^{\mathcal{C}_\gamma / \mathcal{C}_\nu}$; the superscript is meant to invoke the set difference between the two cohorts.

The person-time shared between cohorts $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$ can be divided into either 4 or 5 mutually disjoint "windows" depending on the size of Δ relative to T_{pre} and T_{post}. These windows are based on the cohort's treatment status at a given time. There are 7 possible windows, of which at most 5 can be present in a particular configuration of T_{pre}, T_{post}, and Δ (this is easily seen by inspecting the "Time Duration" column of table 3). As an example, we denote

Table 3: Descriptions of the 11 possible disjoint subdivisions ("windows") of person-time contributed by control individuals in state $\zeta \in \Xi_{\text{ctrl}}$ to either cohort $\mathcal{C}_\gamma$ or $\mathcal{C}_\nu$, where $\gamma, \nu \in \Xi_{\text{tx}}$ and $t_{1\gamma} - t_{1\nu} = \Delta > 0$. Note that $(x)_+ := \max(x, 0)$.

Window	Definition	Time Duration
$\mathcal{D}_{\text{pre}}^{\mathcal{C}_\gamma \backslash \mathcal{C}_\nu}(\zeta)$	Control person-time in state ζ contributing *only* to $\mathcal{C}_\gamma$ in its pre-treatment period. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \backslash \mathcal{I}_{\mathcal{C}_\nu}(\zeta), t < t_{*\gamma}\right\}$	T_{pre}
$\mathcal{D}_{\text{pre}}^{\mathcal{C}_\nu \backslash \mathcal{C}_\gamma}(\zeta)$	Control person-time in state ζ contributing *only* to $\mathcal{C}_\nu$ in its pre-treatment period. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\nu}(\zeta) \backslash \mathcal{I}_{\mathcal{C}_\gamma}(\zeta), t < t_{*\nu}\right\}$	T_{pre}
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma \backslash \mathcal{C}_\nu}(\zeta)$	Control person-time in state ζ contributing *only* to $\mathcal{C}_\gamma$ in its post-treatment period. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \backslash \mathcal{I}_{\mathcal{C}_\nu}(\zeta), t \geq t_{*\gamma}\right\}$	T_{post}
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu \backslash \mathcal{C}_\gamma}(\zeta)$	Control person-time in state ζ contributing *only* to $\mathcal{C}_\nu$ in its post-treatment period. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\nu}(\zeta) \backslash \mathcal{I}_{\mathcal{C}_\gamma}(\zeta), t \geq t_{*\nu}\right\}$	T_{post}
$\mathcal{O}_{\text{pre},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	Person-time from shared controls in state ζ contributing to $\mathcal{C}_\gamma$ in its pre-treatment period while $\mathcal{C}_\nu$'s study period has not started. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \cap \mathcal{I}_{\mathcal{C}_\nu}(\zeta), t < t_{1\nu}\right\}$	$\min\left(T_{\text{pre}}, \Delta\right)$
$\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	Person-time from shared controls in state ζ contributing to both $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$ in their pre-treatment periods. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \cap \mathcal{I}_{\mathcal{C}_\nu}(\zeta), t_{1\nu} \leq t < t_{*\gamma}\right\}$	$\left(T_{\text{pre}} - \Delta\right)_+$
$\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	Person-time from shared controls in state ζ contributing to $\mathcal{C}_\gamma$ in its post-treatment period while $\mathcal{C}_\nu$'s study period has not started. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \cap \mathcal{I}_{\mathcal{C}_\nu}(\zeta), t_{*\gamma} \leq t < t_{1\nu}\right\}$	$\min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right)$
$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	Person-time from shared controls in state ζ contributing to $\mathcal{C}_\gamma$ in its post-treatment period and $\mathcal{C}_\nu$ in its pre-treatment period. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \cap \mathcal{I}_{\mathcal{C}_\nu}(\zeta), \max\left(t_{*\gamma}, t_{1\nu}\right) \leq t < \min\left(t_{T\gamma}, t_{*\nu}\right)\right\}$	$\min\Big(T_{\text{pre}}, T_{\text{post}}, \Delta, \left(T_{\text{pre}} + T_{\text{post}} - \Delta\right)_+\Big)$
$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	Person-time from shared controls in state ζ contributing to both $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$ in their post-treatment periods. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \cap \mathcal{I}_{\mathcal{C}_\nu}(\zeta), t_{*_\nu} \leq t \leq t_{*\gamma}\right\}$	$\left(T_{\text{post}} - \Delta\right)_+$
$\mathcal{O}_{\cdot,\text{pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	Person-time from shared controls in state ζ contributing to $\mathcal{C}_\nu$ in its pre-treatment period after $\mathcal{C}_\gamma$'s study period has ended. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \cap \mathcal{I}_{\mathcal{C}_\nu}(\zeta), t_{T\gamma} < t < t_{*\nu}\right\}$	$\left(\Delta - T_{\text{post}}\right)_+$
$\mathcal{O}_{\cdot,\text{post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	Person-time from shared controls in state ζ contributing to $\mathcal{C}_\nu$ in its post-treatment period after $\mathcal{C}_\gamma$'s study period has ended. $\left\{(i,t) : i \in \mathcal{I}_{\mathcal{C}_\gamma}(\zeta) \cap \mathcal{I}_{\mathcal{C}_\nu}(\zeta), \max\left(t_{T\gamma} + 1, t_{*\nu}\right) \leq t \leq t_{T\nu}\right\}$	$\min\left(T_{\text{post}}, \Delta\right)$

Table 4: Windows of person-time composing each pre- and post-treatment mean in equation (33).

Mean	Description	Component Windows
$\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}}$	Mean over control individuals in $\mathcal{C}_\gamma$ in its post-treatment period.	$\bigcup_{\zeta\in\Xi_{\text{ctrl}}}\left(\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}(\zeta)\cup\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\cup\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\cup\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\right)$
$\bar{Y}_{\text{ctrl},\{t\geq t_{*\nu}\}}$	Mean over control individuals in $\mathcal{C}_\nu$ in its post-treatment period.	$\bigcup_{\zeta\in\Xi_{\text{ctrl}}}\left(\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu\setminus\mathcal{C}_\gamma}(\zeta)\cup\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\cup\mathcal{O}_{\cdot,\text{post}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\right)$
$\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}$	Mean over control individuals in $\mathcal{C}_\gamma$ in its pre-treatment period.	$\bigcup_{\zeta\in\Xi_{\text{ctrl}}}\left(\mathcal{D}_{\text{pre}}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}(\zeta)\cup\mathcal{O}_{\text{pre},\cdot}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\cup\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}\right)$
$\bar{Y}_{\text{ctrl},\{t<t_{*\nu}\}}$	Mean over control individuals in $\mathcal{C}_\nu$ in its pre-treatment period.	$\bigcup_{\zeta\in\Xi_{\text{ctrl}}}\left(\mathcal{D}_{\text{pre}}^{\mathcal{C}_\nu\setminus\mathcal{C}_\gamma}(\zeta)\cup\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\cup\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\cup\mathcal{O}_{\cdot,\text{pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)\right)$

the set of *overlapping* control person-time when $\mathcal{C}_\gamma$ is in its post-treatment period and $\mathcal{C}_\nu$ is in its pre-treatment period as $\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}$. In this notation, we use $\cdot$ to denote a period in which one of the cohorts' study periods has not started or has ended. Figure 8 shows three configurations of T_{pre}, T_{post}, and Δ along with the person-time windows induced by each configuration.

Following the notation in table 3, we write each mean in equation (33) as an average over the person-time that contributes to that mean. For example, in the top diagram in figure 8, with $\Delta = 3$, the pre-treatment mean outcome in the control individuals in cohort $\mathcal{C}_\gamma$, $\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}}$, is an average over $\cup_{\zeta\in\Xi_{\text{ctrl}}}\mathcal{D}_{\text{pre}}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}(\zeta)$, $\cup_{\zeta\in\Xi_{\text{ctrl}}}\mathcal{O}_{\text{pre},\cdot}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)$, and $\cup_{\zeta\in\Xi_{\text{ctrl}}}\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)$. More specifically, we can write

$$\bar{Y}_{\text{ctrl},\{t<t_{*\gamma}\}} = \frac{1}{T_{\text{pre}}N_\gamma^{\text{ctrl}}}\sum_{\zeta\in\Xi_{\text{ctrl}}}\left[\sum_{(i,t)\in\mathcal{D}_{\text{pre}}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}(\zeta)}Y_{\zeta it}+\sum_{(i,t)\in\mathcal{O}_{\text{pre},\cdot}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)}Y_{\zeta it}+\sum_{\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)}Y_{\zeta it}\right]. \tag{34}$$

Similar expansions of each mean in the difference-in-differences estimator in equation (19) can be written over each of their component person-time windows listed in table 4.

Start by considering the first summand on the right-hand side of equation (33), the covariance between post-treatment means among control individuals in cohorts $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$. When we expand each of these means as in equation (34) and use bilinearity of covariance, we can write this as a sum of 12 covariances:

$$\begin{aligned}
&\text{Cov}\left(\bar{Y}_{\text{ctrl},\{t\geq t_{*\gamma}\}},\bar{Y}_{\text{ctrl},\{t\geq t_{*\nu}\}}\right) = \\
&\quad\left(N_\gamma^{\text{ctrl}}N_\nu^{\text{ctrl}}T_{\text{post}}^2\right)^{-1}\left[\text{Cov}\left(\sum_{(i,t)\in\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}(\zeta)}Y_{\zeta it},\sum_{(i,t)\in\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu\setminus\mathcal{C}_\gamma}(\zeta)}Y_{\zeta it}\right)+\text{Cov}\left(\sum_{(i,t)\in\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}(\zeta)}Y_{\zeta it},\sum_{(i,t)\in\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)}Y_{\zeta it}\right)\right. \\
&\qquad+\text{Cov}\left(\sum_{(i,t)\in\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}(\zeta)}Y_{\zeta it},\sum_{(i,t)\in\mathcal{O}_{\cdot,\text{post}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)}Y_{\zeta it}\right)+\text{Cov}\left(\sum_{(i,t)\in\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)}Y_{\zeta it},\sum_{(i,t)\in\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu\setminus\mathcal{C}_\gamma}(\zeta)}Y_{\zeta it}\right) \\
&\qquad\left.+\text{Cov}\left(\sum_{(i,t)\in\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)}Y_{\zeta it},\sum_{(i,t)\in\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu\setminus\mathcal{C}_\gamma}(\zeta)}Y_{\zeta it}\right)+\text{Cov}\left(\sum_{(i,t)\in\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}(\zeta)}Y_{\zeta it},\sum_{(i,t)\in\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu\setminus\mathcal{C}_\gamma}(\zeta)}Y_{\zeta it}\right)\right]
\end{aligned}$$

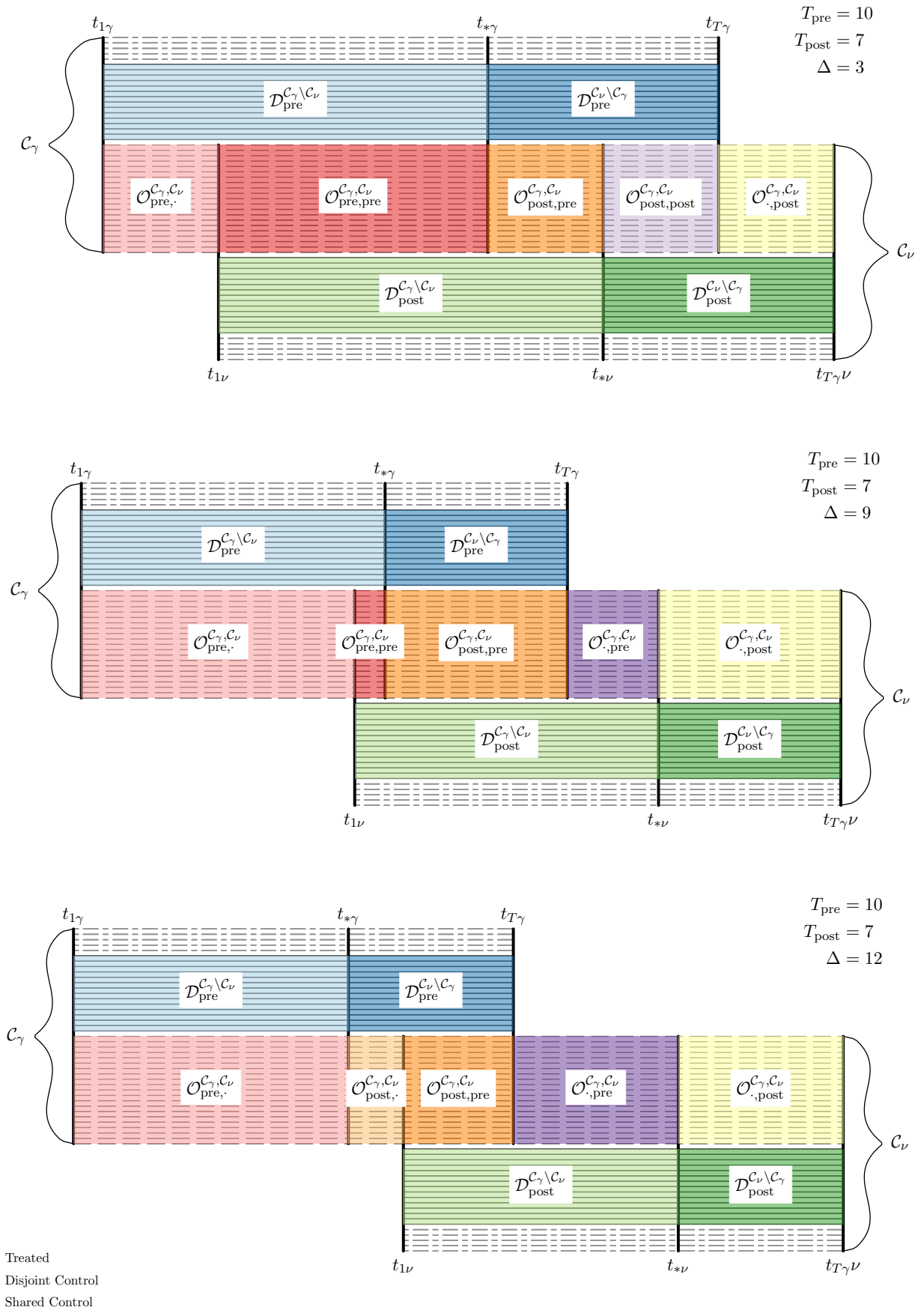

Figure 8: Schematic depictions of overlapping cohorts with varying Δ, highlighted to show person-time windows defined in table 3. The value of Δ relative to T_{pre} and T_{post} determines which windows are present in the shared control person-time.

$$
\begin{aligned}
&+ \operatorname{Cov}\left(\sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}_{\text{post},\cdot}(\zeta)} Y_{\zeta it}, \sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\nu,\mathcal{C}_\gamma}_{\text{post,post}}(\zeta)} Y_{\zeta it} \right) + \operatorname{Cov}\left(\sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}_{\text{post},\cdot}(\zeta)} Y_{\zeta it}, \sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\nu,\mathcal{C}_\gamma}_{\cdot,\text{post}}(\zeta)} Y_{\zeta it} \right) \\
&+ \operatorname{Cov}\left(\sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}_{\text{post,pre}}(\zeta)} Y_{\zeta it}, \sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\nu,\mathcal{C}_\gamma}_{\text{post,post}}(\zeta)} Y_{\zeta it} \right) + \operatorname{Cov}\left(\sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}_{\text{post,pre}}(\zeta)} Y_{\zeta it}, \sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\nu,\mathcal{C}_\gamma}_{\cdot,\text{post}}(\zeta)} Y_{\zeta it} \right) \\
&+ \operatorname{Var}\left(\sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}_{\text{post,post}}(\zeta)} Y_{\zeta it} \right) + \operatorname{Cov}\left(\sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}_{\text{post,post}}(\zeta)} Y_{\zeta it}, \sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\nu,\mathcal{C}_\gamma}_{\cdot,\text{post}}(\zeta)} Y_{\zeta it} \right) \Bigg] . \qquad (35)
\end{aligned}
$$

We now handle each of these covariances in turn. Under a block-exchangeable covariance structure for the data (equation (7)), deriving an expression for each of these component covariances becomes a simple counting problem. For each pair of person-time windows, we simply count the amount of person-time shared within the same individual, shared within the same time, and shared across time. Let $N_{\gamma\setminus\nu}(\zeta)$ be the number of disjoint individuals in state ζ that contribute to $\mathcal{C}_\gamma$ but not $\mathcal{C}_\nu$, and $N_{\gamma\cap\nu}(\zeta)$ the number of control individuals in state ζ shared between $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$. Let $T_{\text{post,post}} := (T_{\text{post}} - \Delta)_+$ be the number of measurement ocassions that contribute to the post-treatment periods of both $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$, and similarly for all other combinations of pre- and post-treatment periods in table 3.

As in **??**, we can write each of the component covariances above into sums of covariances betweeen outcomes in the same person at the same time, the same person at different times, different people at the same times, and different people at different times. As an example, between person time in $\mathcal{D}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}_{\text{post}}$ and person time in $\mathcal{O}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}_{\text{post,post}}$, there are $N_{\gamma\setminus\nu}N_{\gamma\cap\nu}$ combinations of distinct individuals (i.e., no within-individual combinations), each of which containing T_{post} combinations of time. Of those time combinations, $T_{\text{post,post}}$ are between the same time and $T^2_{\text{post}} - T_{\text{post,post}}$ are between different times. Therefore, under equation (7),

$$
\operatorname{Cov}\left(\sum_{(i,t)\in\mathcal{D}^{\mathcal{C}_\gamma\setminus\mathcal{C}_\nu}_{\text{post}}(\zeta)} Y_{\zeta it}, \sum_{(i,t)\in\mathcal{O}^{\mathcal{C}_\gamma,\mathcal{C}_\nu}_{\text{post,post}}(\zeta)} Y_{\zeta it} \right) = N_{\gamma\setminus\nu}N_{\gamma\cap\nu}\left(T_{\text{post,post}}\phi_\zeta + \left(T^2_{\text{post}} - T_{\text{post,post}}\right)\psi_\zeta\right)\sigma^2_\zeta . \qquad (36)
$$

Using the same reasoning, we can find expressions for every covariance on the right-hand side of equation (35). We provide them in table 5. Similarly, we provide expressions for all component covariances of the remaining terms on the right-hand side of equation (33) in tables 6 to 8.

A closed-form expression of $\operatorname{Cov}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right)$ can be derived as follows:

$$
\begin{aligned}
\operatorname{Cov}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right) = &\left(N^{\text{ctrl}}_\gamma N^{\text{ctrl}}_\nu T^2_{\text{post}}\right)^{-1} \times (\text{sum of expressions in table 5}) \\
&+ \left(N^{\text{ctrl}}_\gamma N^{\text{ctrl}}_\nu T^2_{\text{pre}}\right)^{-1} \times (\text{sum of expressions in table 6}) \\
&- \left(N^{\text{ctrl}}_\gamma N^{\text{ctrl}}_\nu T_{\text{post}}T_{\text{pre}}\right)^{-1} \times (\text{sum of expressions in table 7}) \\
&- \left(N^{\text{ctrl}}_\gamma N^{\text{ctrl}}_\nu T_{\text{pre}}T_{\text{post}}\right)^{-1} \times (\text{sum of expressions in table 8}) .
\end{aligned}
$$

Table 5: Closed-form expressions of component covariances on the right-hand side of equation (35), the covariance of post-treatment means over control individuals for cohorts $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$.

X_1	X_2	$\mathrm{Cov}\left(\sum_{(i,t)\in X_1} Y_{\zeta it}, \sum_{(i,t)\in X_2} Y_{\zeta it}\right)$
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\nu\setminus\gamma}(\zeta) \left(T_{\text{post,post}} \phi_\zeta + \left(T_{\text{post}}^2 - T_{\text{post,post}}\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{post,post}} \left(\phi_\zeta + \left(T_{\text{post}} - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\text{post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{post}} T_{\cdot,\text{post}} \psi_\zeta \sigma_\zeta^2$
$\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{post},\cdot} T_{\text{post}} \psi_\zeta \sigma_\zeta^2$
$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{post}} T_{\text{post,pre}} \psi_\zeta \sigma_\zeta^2$
$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\text{post}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) \left(T_{\text{post,post}} \phi_\zeta T_{\text{post,post}} \left(T_{\text{post}} - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post},\cdot} T_{\text{post,post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\text{post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post},\cdot} T_{\cdot,\text{post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,pre}} T_{\text{post,post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\text{post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,pre}} T_{\cdot,\text{post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,post}} \left(1 + \left(T_{\text{post,post}} - 1\right) \rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \left(\phi_\zeta + \left(T_{\text{post,post}} - 1\right) \psi_\zeta\right)\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\text{post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,post}} T_{\cdot,\text{post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$

Table 6: Closed-form expressions of component covariances of the covariance of pre-treatment means over control individuals for cohorts $\mathcal{C}_\gamma$ and $\mathcal{C}_\nu$.

X_1	X_2	$\mathrm{Cov}\left(\sum_{(i,t)\in X_1} Y_{\zeta it}, \sum_{(i,t)\in X_2} Y_{\zeta it}\right)$
$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta)N_{\nu\setminus\gamma}(\zeta)\left(T_{\mathrm{pre,pre}}\phi_\zeta + \left(T_{\mathrm{pre}}^2 - T_{\mathrm{pre,pre}}\right)\psi_\zeta\right)\sigma_\zeta^2$
$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta)N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre,pre}}\left(\phi_\zeta + \left(T_{\mathrm{pre}} - 1\right)\psi_\zeta\right)\sigma_\zeta^2$
$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta)N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre}}T_{\mathrm{post,pre}}\psi_\zeta\sigma_\zeta^2$
$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\mathrm{pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta)N_{\gamma\cap\nu}(\zeta)T_{\cdot,\mathrm{pre}}T_{\mathrm{pre}}\psi_\zeta\sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\nu\setminus\gamma}(\zeta)N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre}}T_{\mathrm{pre},\cdot}\psi_\zeta\sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\nu\setminus\gamma}(\zeta)N_{\gamma\cap\nu}(\zeta)\left(T_{\mathrm{pre,pre}}\phi_\zeta T_{\mathrm{pre,pre}}\left(T_{\mathrm{pre}} - 1\right)\psi_\zeta\right)\sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre},\cdot}T_{\mathrm{pre,pre}}\left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right)\psi_\zeta\right)\sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre},\cdot}T_{\mathrm{post,pre}}\left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right)\psi_\zeta\right)\sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\mathrm{pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre},\cdot}T_{\cdot,\mathrm{pre}}\left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right)\psi_\zeta\right)\sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre,pre}}\left(1 + \left(T_{\mathrm{pre,pre}} - 1\right)\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right)\left(\phi_\zeta\ + \left(T_{\mathrm{pre,pre}} - 1\right)\psi_\zeta\right)\right)\sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre,pre}}T_{\mathrm{post,pre}}\left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right)\psi_\zeta\right)\sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\mathrm{pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta)T_{\mathrm{pre,pre}}T_{\cdot,\mathrm{pre}}\left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right)\psi_\zeta\right)\sigma_\zeta^2$

Table 7: Closed-form expressions of component covariances of the covariance between the post-treatment mean over control individuals for cohort $\mathcal{C}_\gamma$ and the pre-treatment mean over control individuals for cohort $\mathcal{C}_\nu$. Note that some covariances are identically zero because the two windows cannot both have non-zero duration; see table 3.

X_1	X_2	$\mathrm{Cov}\left(\sum_{(i,t)\in X_1} Y_{\zeta it}, \sum_{(i,t)\in X_2} Y_{\zeta it}\right)$
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\text{pre}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\nu\setminus\gamma}(\zeta) \left(T_{\text{post,pre}} \phi_\zeta + \left(T_{\text{pre}} T_{\text{post}} - T_{\text{post,pre}}\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\text{pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{post}} T_{\cdot,\text{pre}} \left(\phi_\zeta + \left(T_{\text{pre}} - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{post,pre}} \left(\phi_\zeta + \left(T_{\text{post}} - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{D}_{\text{post}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{post}} T_{\text{pre,pre}} \psi_\zeta \sigma_\zeta^2$
$\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\text{pre}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\nu\setminus\gamma}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{pre}} T_{\text{post},\cdot} \psi_\zeta \sigma_\zeta^2$
$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\text{pre}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\nu\setminus\gamma}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\text{post,pre}} \left(\phi_\zeta + \left(T_{\text{pre}} - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\text{pre}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\nu\setminus\gamma}(\zeta) N_{\gamma\cap\nu}(\zeta) \left(T_{\text{post,post}} T_{\text{pre}} \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post},\cdot} T_{\text{pre,pre}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2 \equiv 0$
$\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post},\cdot} T_{\text{post,pre}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\text{pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post},\cdot} T_{\cdot,\text{pre}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\text{pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,pre}} T_{\cdot,\text{pre}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,pre}} T_{\text{pre,pre}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right)$
$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,pre}} \left(1 + \left(T_{\text{post,post}} - 1\right) \rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \left(\phi_\zeta \ + \left(T_{\text{post,pre}} - 1\right) \psi_\zeta\right)\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,post}} T_{\text{pre,pre}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\text{pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	0
$\mathcal{O}_{\text{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\text{post,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\text{post,post}} T_{\text{post,pre}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$

Table 8: Closed-form expressions of component covariances of the covariance between the pre-treatment mean over control individuals for cohort $\mathcal{C}_\gamma$ and the post-treatment mean over control individuals for cohort $\mathcal{C}_\nu$. Note that some covariances are identically zero because the two windows cannot both have non-zero duration; see table 3.

X_1	X_2	$\mathrm{Cov}\left(\sum_{(i,t)\in X_1} Y_{\zeta it}, \sum_{(i,t)\in X_2} Y_{\zeta it}\right)$
$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\mathrm{post}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\nu\setminus\gamma}(\zeta) T_{\mathrm{pre}} T_{\mathrm{post}} \psi_\zeta \sigma_\zeta^2$
$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\mathrm{pre}} T_{\mathrm{post,post}} \psi_\zeta \sigma_\zeta^2$
$\mathcal{D}_{\mathrm{pre}}^{\mathcal{C}_\gamma \setminus \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\mathrm{post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\setminus\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\mathrm{pre}} T_{\cdot,\mathrm{pre}} \psi_\zeta \sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\mathrm{post}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\nu\setminus\gamma}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\mathrm{post}} T_{\mathrm{pre},\cdot} \psi_\zeta \sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{D}_{\mathrm{post}}^{\mathcal{C}_\nu \setminus \mathcal{C}_\gamma}(\zeta)$	$N_{\nu\setminus\gamma}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\mathrm{post}} T_{\mathrm{pre,pre}} \psi_\zeta \sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\mathrm{pre},\cdot} T_{\mathrm{post,post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\mathrm{post,post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) N_{\gamma\cap\nu}(\zeta) T_{\mathrm{pre,pre}} T_{\mathrm{post,post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$
$\mathcal{O}_{\mathrm{pre},\cdot}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\mathrm{post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\mathrm{pre},\cdot} T_{\cdot,\mathrm{post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2 \equiv 0$
$\mathcal{O}_{\mathrm{pre,pre}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$\mathcal{O}_{\cdot,\mathrm{post}}^{\mathcal{C}_\gamma, \mathcal{C}_\nu}(\zeta)$	$N_{\gamma\cap\nu}(\zeta) T_{\mathrm{pre,pre}} T_{\cdot,\mathrm{post}} \left(\rho_\zeta + \left(N_{\gamma\cap\nu}(\zeta) - 1\right) \psi_\zeta\right) \sigma_\zeta^2$

Combining terms, we have

$$\begin{aligned}\operatorname{Cov}\left(\widehat{\mathrm{ATT}}_\gamma, \widehat{\mathrm{ATT}}_\nu\right) = (T_{\mathrm{pre}}T_{\mathrm{post}})^{-2}\left(N_\gamma^{\mathrm{ctrl}}N_\nu^{\mathrm{ctrl}}\right)^{-1}\sum_{\zeta\in\Xi_{\mathrm{ctrl}}}\sigma_\zeta^2\big[&N_{\gamma\setminus\nu}(\zeta)N_{\nu\setminus\gamma}(\zeta)h_1(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta,\rho_\zeta,\phi_\zeta,\psi_\zeta)\\ &+N_{\gamma\cap\nu}(\zeta)N_{\gamma\setminus\nu}(\zeta)h_2(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta,\rho_\zeta,\phi_\zeta,\psi_\zeta)\\ &+N_{\gamma\cap\nu}(\zeta)N_{\nu\setminus\gamma}(\zeta)h_3(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta,\rho_\zeta,\phi_\zeta,\psi_\zeta)\\ &+N_{\gamma\cap\nu}(\zeta)h_4(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta,\rho_\zeta,\phi_\zeta,\psi_\zeta)\\ &+N_{\gamma\cap\nu}(\zeta)\left(N_{\gamma\cap\nu}(\zeta)-1\right)h_5(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta,\rho_\zeta,\phi_\zeta,\psi_\zeta)\big].\end{aligned} \tag{37}$$

We turn now to simplifying the $h(\cdot)$ functions above. Let $(x)_+ := \max(x,0)$ and define

$$\min(\cdots) := \min\left(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta, (T_{\mathrm{pre}}+T_{\mathrm{post}}-\Delta)_+\right).$$

Collecting terms and substituting time durations from table 3 – such that, e.g., $T_{\mathrm{post,post}} = (T_{\mathrm{post}}-\Delta)_+$) – we have

$$\begin{aligned}h_1(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta,\rho_\zeta,\phi_\zeta,\psi_\zeta) &= T_{\mathrm{pre}}^2(T_{\mathrm{post}}-\Delta)_+\phi_\zeta + T_{\mathrm{pre}}^2\left(T_{\mathrm{post}}^2-(T_{\mathrm{post}}-\Delta)_+\right)\psi_\zeta + T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)_+\phi_\zeta\\ &\quad + T_{\mathrm{post}}^2\left(T_{\mathrm{pre}}^2-(T_{\mathrm{pre}}-\Delta)_+\right)\psi_\zeta - T_{\mathrm{pre}}T_{\mathrm{post}}\min(\cdots)\phi_\zeta\\ &\quad - T_{\mathrm{pre}}T_{\mathrm{post}}\left(T_{\mathrm{pre}}T_{\mathrm{post}}-\min(\cdots)\right)\psi_\zeta - T_{\mathrm{pre}}^2T_{\mathrm{post}}^2\psi_\zeta\\ &= \left(T_{\mathrm{pre}}^2(T_{\mathrm{post}}-\Delta)_+ + T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)_+ - T_{\mathrm{pre}}T_{\mathrm{post}}\min(\cdots)\right)(\phi_\zeta-\psi_\zeta)\\ &=: g_1(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta)(\phi_\zeta-\psi_\zeta).\end{aligned} \tag{38}$$

Notice that $\min(T_{\mathrm{post}},\Delta) + (T_{\mathrm{post}}-\Delta)_+ = T_{\mathrm{post}}$; similarly for other such combinations. Then, for h_2 and h_3,

$$\begin{aligned}h_2(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta,\rho_\zeta,\phi_\zeta,\psi_\zeta) &= T_{\mathrm{post}}^2T_{\mathrm{pre}}(\Delta-T_{\mathrm{post}})_+\psi_\zeta + T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)_+\phi_\zeta + T_{\mathrm{post}}(T_{\mathrm{pre}}-\Delta)_+(T_{\mathrm{pre}}-1)\psi_\zeta\\ &\quad + T_{\mathrm{post}}^2\min(\cdots)T_{\mathrm{pre}}\psi_\zeta + T_{\mathrm{pre}}^2T_{\mathrm{post}}\min(T_{\mathrm{post}},\Delta)\psi_\zeta + T_{\mathrm{pre}}^2(T_{\mathrm{post}}-\Delta)_+\phi_t\\ &\quad + T_{\mathrm{pre}}^2(T_{\mathrm{post}}-\Delta)_+(T_{\mathrm{post}}-1)\psi_\zeta - T_{\mathrm{pre}}T_{\mathrm{post}}^2(\Delta-T_{\mathrm{post}})_+\psi_\zeta - T_{\mathrm{pre}}T_{\mathrm{post}}\min(\cdots)\phi_\zeta\\ &\quad - T_{\mathrm{pre}}T_{\mathrm{post}}\min(\cdots)(T_{\mathrm{post}}-1)\psi_\zeta - T_{\mathrm{pre}}T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)_+\psi_\zeta\\ &\quad - T_{\mathrm{pre}}^2T_{\mathrm{post}}\min(T_{\mathrm{post}},\Delta_+)\psi_\zeta - T_{\mathrm{pre}}^2T_{\mathrm{post}}(T_{\mathrm{post}}-\Delta)_+\psi_\zeta\\ &= \left(T_{\mathrm{pre}}(T_{\mathrm{post}}-\Delta)_+ + T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)_+ - T_{\mathrm{pre}}T_{\mathrm{post}}\min(\cdots)\right)(\phi_\zeta-\psi_\zeta)\\ &=: g_1(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta)(\phi_\zeta-\psi_\zeta).\end{aligned} \tag{39}$$

$$\begin{aligned}h_3(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta,\rho_\zeta,\phi_\zeta,\psi_\zeta) &= T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)\phi_\zeta + T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)_+(T_{\mathrm{pre}}-1)\psi_\zeta + T_{\mathrm{pre}}T_{\mathrm{post}}^2\min(T_{\mathrm{pre}},\Delta)\psi_\zeta\\ &\quad + T_{\mathrm{pre}}^2T_{\mathrm{post}}\min\left((\Delta-T_{\mathrm{pre}})_+,T_{\mathrm{post}}\right)\psi_\zeta + T_{\mathrm{pre}}^2T_{\mathrm{post}}\min(\cdots)\psi_\zeta + T_{\mathrm{pre}}^2(T_{\mathrm{post}}-\Delta)_+\phi_\zeta\\ &\quad + T_{\mathrm{pre}}^2(T_{\mathrm{post}}-\Delta)_+(T_{\mathrm{post}}-1)\psi_\zeta - T_{\mathrm{pre}}^2T_{\mathrm{post}}\min\left((\Delta-T_{\mathrm{pre}})_+,T_{\mathrm{post}}\right)\psi_\zeta\\ &\quad - T_{\mathrm{pre}}T_{\mathrm{post}}\min(\cdots)\phi_\zeta - T_{\mathrm{pre}}T_{\mathrm{post}}\min(\cdots)(T_{\mathrm{pre}}-1)\psi_\zeta - T_{\mathrm{pre}}T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)_+\psi_\zeta\\ &\quad - T_{\mathrm{pre}}T_{\mathrm{post}}^2\min(T_{\mathrm{pre}},\Delta)\psi_\zeta\\ &= \left(T_{\mathrm{pre}}(T_{\mathrm{post}}-\Delta)_+ + T_{\mathrm{post}}^2(T_{\mathrm{pre}}-\Delta)_+ - T_{\mathrm{pre}}T_{\mathrm{post}}\min(\cdots)\right)(\phi_\zeta-\psi_\zeta)\\ &=: g_1(T_{\mathrm{pre}},T_{\mathrm{post}},\Delta)(\phi_\zeta-\psi_\zeta).\end{aligned} \tag{40}$$

Expressions for h_1, h_2, and h_3 have all been easily reached simply by cancelling terms. For h_4, we have

$$\begin{aligned}
h_4\left(T_{\text{pre}}, T_{\text{post}}, \Delta, \rho_\zeta, \phi_\zeta, \psi_\zeta\right) = {} & T_{\text{post}}^2 \min\left(T_{\text{pre}}, \Delta\right)\left(T_{\text{pre}} - \Delta\right)_+ \rho_\zeta + T_{\text{post}}^2 \min\left(T_{\text{pre}}, \Delta\right) \min\left(\cdots\right) \rho_\zeta + T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ \\
& + T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ \left(\left(T_{\text{pre}} - \Delta\right)_+ - 1\right) \rho_\zeta + T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ \left(\Delta - T_{\text{post}}\right)_+ \rho_\zeta \\
& + T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ \min\left(\cdots\right) \rho_\zeta + T_{\text{post}}^2 \min\left(T_{\text{pre}}, \Delta\right)\left(\Delta - T_{\text{post}}\right)_+ \rho_\zeta \\
& + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ \min\left(\cdots\right) \rho_\zeta + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ \\
& + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ \left(\left(T_{\text{post}} - \Delta\right)_+ - 1\right) \rho_\zeta + T_{\text{pre}}^2 \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right)\left(T_{\text{post}} - \Delta\right)_+ \rho_\zeta \\
& + T_{\text{pre}}^2 \min\left(T_{\text{post}}, \Delta\right) \min\left(\cdots\right) \rho_\zeta + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ \min\left(T_{\text{post}}, \Delta\right) \rho_\zeta \\
& + T_{\text{pre}}^2 \min\left(T_{\text{post}}, \Delta\right) \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) \rho_\zeta - T_{\text{pre}} T_{\text{post}} \left(T_{\text{pre}} - \Delta\right)_+ \min\left(\cdots\right) \rho_\zeta \\
& - T_{\text{pre}} T_{\text{post}} \left(T_{\text{post}} - \Delta\right)_+ \left(T_{\text{pre}} - \Delta\right)_+ \rho_\zeta - T_{\text{pre}} T_{\text{post}} \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) \min\left(\cdots\right) \rho_\zeta \\
& - T_{\text{pre}} T_{\text{post}} \min\left(\cdots\right) - T_{\text{pre}} T_{\text{post}} \min\left(\cdots\right)\left(\min\left(\cdots\right) - 1\right) \rho_\zeta \\
& - T_{\text{pre}} T_{\text{post}} \left(T_{\text{post}} - \Delta\right)_+ \min\left(\cdots\right) \rho_\zeta - T_{\text{pre}} T_{\text{post}} \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right)\left(\Delta - T_{\text{post}}\right)_+ \rho_\zeta \\
& - T_{\text{pre}} T_{\text{post}} \min\left(\cdots\right)\left(\Delta - T_{\text{post}}\right)_+ \rho_\zeta - T_{\text{pre}} T_{\text{post}} \min\left(T_{\text{pre}}, \Delta\right)\left(T_{\text{post}} - \Delta\right) \rho_\zeta \\
& - T_{\text{pre}} T_{\text{post}} \left(T_{\text{post}} - \Delta\right)_+ \left(T_{\text{pre}} - \Delta\right)_+ \rho_\zeta - T_{\text{pre}} T_{\text{post}} \min\left(T_{\text{pre}}, \Delta\right) \min\left(T_{\text{post}}, \Delta\right) \rho_\zeta \\
& - T_{\text{pre}} T_{\text{post}} \left(T_{\text{pre}} - \Delta\right)_+ \min\left(T_{\text{post}}, \Delta\right) \rho_\zeta.
\end{aligned}$$

We simplify this by again recognizing that $\min\left(T_{\text{post}}, \Delta\right) + \left(T_{\text{post}} - \Delta\right)_+ = T_{\text{post}}$, etc. After some algebra, we have

$$\begin{aligned}
h_4\left(\sim\right) = {} & \left(1 - \rho_\zeta\right)\left[T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ + T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ - T_{\text{pre}} T_{\text{post}} \min\left(\cdots\right)\right] \\
& + \rho_\zeta T_{\text{pre}} T_{\text{post}} \left\{T_{\text{post}} \left(\Delta - T_{\text{post}}\right)_+ + T_{\text{pre}} \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right)\right. \\
& \qquad - \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right)\left(\Delta - T_{\text{post}}\right)_+ - \min(T_{\text{pre}}, \Delta) \min(T_{\text{post}}, \Delta) \\
& \qquad + \min(\cdots)\left[T_{\text{pre}} + T_{\text{post}} - \left(T_{\text{pre}} - \Delta\right)_+ - \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right)\right. \\
& \qquad\qquad \left.\left. - \left(T_{\text{post}} - \Delta\right)_+ - \left(\Delta - T_{\text{post}}\right)_+ - \min(\cdots)\right]\right\}. \\
=: {} & \left(1 - \rho_\zeta\right) g_1\left(T_{\text{pre}}, T_{\text{post}}, \Delta\right) + \rho_\zeta g_2\left(T_{\text{pre}}, T_{\text{post}}, \Delta\right) \qquad (41)
\end{aligned}$$

We now attempt to understand $g_2\left(T_{\text{pre}}, T_{\text{post}}, \Delta\right)$ in equation (41). We consider various values of Δ relative to T_{pre} and T_{post}. Note that if $\Delta < T_{\text{pre}} + T_{\text{post}}$ then $\min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) = \left(\Delta - T_{\text{pre}}\right)_+$.

- If $\Delta < \min(T_{\text{pre}}, T_{\text{post}})$, then $\min\left(T_{\text{pre}}, T_{\text{post}}, \Delta, \left(T_{\text{pre}} + T_{\text{post}} - \Delta\right)_+\right) = \Delta$ and therefore

$$\begin{aligned}
g_2\left(\sim\right) = {} & T_{\text{pre}} T_{\text{post}} \left\{T_{\text{post}} \cdot 0 + T_{\text{pre}} \cdot 0 - 0 \cdot 0 - \Delta^2\right. \\
& \qquad \left. + \Delta\left[T_{\text{pre}} + T_{\text{post}} - \left(T_{\text{pre}} - \Delta\right) - 0 - \left(T_{\text{post}} - \Delta\right) - 0 - \Delta\right]\right\} \\
= {} & 0.
\end{aligned}$$

- If $T_{\text{pre}} < \Delta < T_{\text{post}}$, then $\min\left(T_{\text{pre}}, T_{\text{post}}, \Delta, \left(T_{\text{pre}} + T_{\text{post}} - \Delta\right)_+\right) = T_{\text{pre}}$ and therefore

$$\begin{aligned}
g_2\left(\sim\right) = {} & T_{\text{pre}} T_{\text{post}} \left\{T_{\text{post}} \cdot 0 + T_{\text{pre}}\left(\Delta - T_{\text{pre}}\right) - \left(\Delta - T_{\text{pre}}\right) \cdot 0 - T_{\text{pre}} \Delta\right. \\
& \qquad \left. + T_{\text{pre}}\left[T_{\text{pre}} + T_{\text{post}} - 0 - \left(\Delta - T_{\text{pre}}\right) - \left(T_{\text{post}} - \Delta\right) - 0 - T_{\text{pre}}\right]\right\} \\
= {} & 0.
\end{aligned}$$

- If $T_{\text{post}} < \Delta < T_{\text{pre}}$, then $\min\left(T_{\text{pre}}, T_{\text{post}}, \Delta, \left(T_{\text{pre}} + T_{\text{post}} - \Delta\right)_+\right) = T_{\text{pre}}$ and therefore

$$\begin{aligned}
g_2\left(\sim\right) = {} & T_{\text{pre}} T_{\text{post}} \left\{T_{\text{post}} \cdot 0 + T_{\text{pre}}\left(\Delta - T_{\text{pre}}\right) - \left(\Delta - T_{\text{pre}}\right) \cdot 0 - T_{\text{pre}} \Delta\right. \\
& \qquad \left. + T_{\text{pre}}\left[T_{\text{pre}} + T_{\text{post}} - 0 - \left(\Delta - T_{\text{pre}}\right) - \left(T_{\text{post}} - \Delta\right) - 0 - T_{\text{pre}}\right]\right\} \\
= {} & 0.
\end{aligned}$$

- For $\max(T_{\text{pre}}, T_{\text{post}}) < \Delta < T_{\text{pre}} + T_{\text{post}}$, we have $\min\left(T_{\text{pre}}, T_{\text{post}}, \Delta, \left(T_{\text{pre}} + T_{\text{post}} - \Delta\right)_+\right) = T_{\text{pre}} + T_{\text{post}} - \Delta$

and therefore

$$\begin{aligned}
g_2(\sim) &= T_{\text{pre}}T_{\text{post}}\{T_{\text{post}}(\Delta - T_{\text{post}}) + T_{\text{pre}}(\Delta - T_{\text{pre}}) - (\Delta - T_{\text{pre}})(\Delta - T_{\text{post}}) - T_{\text{pre}}T_{\text{post}} \\
&\qquad + (T_{\text{pre}} + T_{\text{post}} - \Delta)\left[T_{\text{pre}} + T_{\text{post}} - 0 - (\Delta - T_{\text{pre}}) - 0 - (\Delta - T_{\text{post}}) - (T_{\text{pre}} + T_{\text{post}} - \Delta)\right]\} \\
&= T_{\text{pre}}T_{\text{post}}\left\{-(T_{\text{pre}} + T_{\text{post}})^2 + 2\Delta(T_{\text{pre}} + T_{\text{post}}) - \Delta^2 + (T_{\text{pre}} + T_{\text{post}} - \Delta)^2\right\} \\
&= 0.
\end{aligned}$$

- For $\max(T_{\text{pre}}, T_{\text{post}}) < T_{\text{pre}} + T_{\text{post}} < \Delta$, we have $\min\left(T_{\text{pre}}, T_{\text{post}}, \Delta, (T_{\text{pre}} + T_{\text{post}} - \Delta)_+\right) = 0$ and, since $\Delta - T_{\text{pre}} > T_{\text{post}}$, $\min\left((\Delta - T_{\text{pre}})_+, T_{\text{post}}\right) = T_{\text{post}}$; therefore

$$\begin{aligned}
g_2(\sim) &= T_{\text{pre}}T_{\text{post}}\{T_{\text{post}}(\Delta - T_{\text{post}}) + T_{\text{pre}}T_{\text{post}} - T_{\text{post}}(\Delta - T_{\text{post}}) - T_{\text{pre}}T_{\text{post}} + 0\} \\
&= 0
\end{aligned}$$

Since we have partitioned the positive real line with the above, $g_2(T_{\text{pre}}, T_{\text{post}}, \Delta) = 0$ for all positive T_{pre}, T_{post}, and Δ, so that

$$\begin{aligned}
h_4(T_{\text{pre}}, T_{\text{post}}, \Delta, \rho_\zeta, \phi_\zeta, \psi_\zeta) &= \left(T_{\text{pre}}^2(T_{\text{post}} - \Delta)_+ + T_{\text{post}}^2(T_{\text{pre}} - \Delta)_+ - T_{\text{pre}}T_{\text{post}}\min(\cdots)\right)(1 - \rho_\zeta) \qquad (42) \\
&= g_1(T_{\text{pre}}, T_{\text{post}}, \Delta)(1 - \rho_\zeta).
\end{aligned}$$

Finally, we turn to h_5:

$$\begin{aligned}
h_5(T_{\text{pre}}, T_{\text{post}}, \Delta, \rho_\zeta, \phi_\zeta, \psi_\zeta) = {} & T_{\text{post}}^2\min(T_{\text{pre}}, \Delta)(T_{\text{pre}} - \Delta)_+\psi_\zeta + T_{\text{post}}^2\min(T_{\text{pre}}, \Delta)\min(\cdots)\psi_\zeta \\
& + T_{\text{post}}^2(T_{\text{pre}} - \Delta)_+\phi_\zeta + T_{\text{post}}^2(T_{\text{pre}} - \Delta)_+\left((T_{\text{pre}} - \Delta)_+ - 1\right)\psi_\zeta \\
& + T_{\text{post}}^2(T_{\text{pre}} - \Delta)_+(\Delta - T_{\text{post}})_+\psi_\zeta + T_{\text{post}}^2(T_{\text{pre}} - \Delta)_+\min(\cdots)\psi_\zeta \\
& + T_{\text{post}}^2\min(T_{\text{pre}}, \Delta)(\Delta - T_{\text{post}})_+\psi_\zeta + T_{\text{pre}}^2\min(\cdots)(T_{\text{post}} - \Delta)_+\psi_\zeta \\
& + T_{\text{pre}}^2(T_{\text{post}} - \Delta)_+\phi_\zeta + T_{\text{pre}}^2((T_{\text{post}} - \Delta) - 1)\psi_\zeta \\
& + T_{\text{pre}}^2\min\left((\Delta - T_{\text{pre}})_+, T_{\text{post}}\right)(T_{\text{post}} - \Delta)_+\psi_\zeta + T_{\text{pre}}^2\min(\cdots)\min(T_{\text{post}}, \Delta)\psi_\zeta \\
& + T_{\text{pre}}^2(T_{\text{post}} - \Delta)_+\min(T_{\text{post}}, \Delta)\psi_\zeta + T_{\text{pre}}^2\min\left((\Delta - T_{\text{pre}})_+, T_{\text{post}}\right)\min(T_{\text{post}}, \Delta)\psi_\zeta \\
& - T_{\text{pre}}T_{\text{post}}\min(\cdots)(T_{\text{pre}} - \Delta)_+\psi_\zeta - T_{\text{pre}}T_{\text{post}}(T_{\text{post}} - \Delta)_+(T_{\text{pre}} - \Delta)_+\psi_\zeta \\
& - T_{\text{pre}}T_{\text{post}}\min\left((\Delta - T_{\text{pre}})_+, T_{\text{post}}\right)\min(\cdots)\psi_\zeta - T_{\text{pre}}T_{\text{post}}\min(\cdots)\phi_\zeta \\
& - T_{\text{pre}}T_{\text{post}}\min(\cdots)(\min(\cdots) - 1)\psi_\zeta - T_{\text{pre}}T_{\text{post}}(T_{\text{post}} - \Delta)_+\min(\cdots)\psi_\zeta \\
& - T_{\text{pre}}T_{\text{post}}\min\left((\Delta - T_{\text{pre}})_+, T_{\text{post}}\right)(\Delta - T_{\text{post}})_+\psi_\zeta - T_{\text{pre}}T_{\text{post}}\min(\cdots)(\Delta - T_{\text{post}})_+\psi_\zeta \\
& - T_{\text{pre}}T_{\text{post}}\min(T_{\text{pre}}, \Delta)(T_{\text{post}} - \Delta)_+\psi_\zeta - T_{\text{pre}}T_{\text{post}}(T_{\text{post}} - \Delta)_+(T_{\text{pre}} - \Delta)_+\psi_\zeta \\
& - T_{\text{pre}}T_{\text{post}}\min(T_{\text{pre}}, \Delta)\min(T_{\text{post}}, \Delta)\psi_\zeta - T_{\text{pre}}T_{\text{post}}(T_{\text{pre}} - \Delta)_+\min(T_{\text{post}}, \Delta)\psi_\zeta.
\end{aligned}$$

Gathering terms as above, we have

$$
\begin{aligned}
h_5(\sim) &= \left(T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ - T_{\text{pre}} T_{\text{post}} \min(\cdots)\right)(\phi_\zeta - \psi_\zeta) \\
&\quad + \psi_\zeta \Big\{ T_{\text{pre}} T_{\text{post}} \min(\cdots) \Big[T_{\text{post}} + T_{\text{pre}} - \left(T_{\text{pre}} - \Delta\right)_+ - \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) - \left(T_{\text{post}} - \Delta\right)_+ \\
&\qquad\qquad - \left(\Delta - T_{\text{post}}\right)_+ - \min(\cdots) \Big] \\
&\qquad + T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ \left(\Delta - T_{\text{post}}\right)_+ + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) \\
&\qquad + T_{\text{post}}^2 \min(T_{\text{pre}}, \Delta) \left(\Delta - T_{\text{post}}\right)_+ + T_{\text{pre}}^2 \min(T_{\text{post}}, \Delta) \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) \\
&\qquad - T_{\text{pre}} T_{\text{post}} \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) \left(\Delta - T_{\text{post}}\right)_+ - T_{\text{pre}} T_{\text{post}} \min(T_{\text{pre}}, \Delta) \min(T_{\text{post}}, \Delta) \Big\} \\
&= \left(T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ - T_{\text{pre}} T_{\text{post}} \min(\cdots)\right)(\phi_\zeta - \psi_\zeta) \\
&\quad + \psi_\zeta T_{\text{pre}} T_{\text{post}} \Big\{ \min(\cdots) \Big[T_{\text{post}} + T_{\text{pre}} - \left(T_{\text{pre}} - \Delta\right)_+ - \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) - \left(T_{\text{post}} - \Delta\right)_+ \\
&\qquad\qquad - \left(\Delta - T_{\text{post}}\right)_+ - \min(\cdots) \Big] \\
&\qquad + T_{\text{post}} \left(\Delta - T_{\text{post}}\right)_+ + T_{\text{pre}} \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) \\
&\qquad - \min\left(\left(\Delta - T_{\text{pre}}\right)_+, T_{\text{post}}\right) \left(\Delta - T_{\text{post}}\right) - \min(T_{\text{pre}}, \Delta) \min(T_{\text{post}}, \Delta) \Big\} \\
&= \left(T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ - T_{\text{pre}} T_{\text{post}} \min(\cdots)\right)(\phi_\zeta - \psi_\zeta) + \psi_\zeta g_2(T_{\text{pre}}, T_{\text{post}}, \Delta) \\
&= \left(T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ - T_{\text{pre}} T_{\text{post}} \min(\cdots)\right)(\phi_\zeta - \psi_\zeta) \qquad (43) \\
&= g_1(T_{\text{pre}}, T_{\text{post}}, \Delta)(\phi_\zeta - \psi_\zeta),
\end{aligned}
$$

since we saw above that $g_2(T_{\text{pre}}, T_{\text{post}}, \Delta)$ is identically zero for non-negative T_{pre}, T_{post}, and Δ.

Now, we substitute equations (38) to (43) back into equation (37):

$$
\begin{aligned}
\text{Cov}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right) &= (T_{\text{pre}} T_{\text{post}})^{-2} \left(N_\gamma^{\text{ctrl}} N_\nu^{\text{ctrl}}\right)^{-1} g_1(T_{\text{pre}}, T_{\text{post}}, \Delta) \\
&\quad \times \sum_{\zeta \in \Xi_{\text{ctrl}}} \Big[(\phi_\zeta - \psi_\zeta) \left(N_{\gamma \setminus \nu}(\zeta) \left(N_{\nu \setminus \gamma}(\zeta) + N_{\gamma \cap \nu}(\zeta)\right) + N_{\gamma \cap \nu}(\zeta) \left(N_{\nu \setminus \gamma}(\zeta) + N_{\gamma \cap \nu}(\zeta) - 1\right)\right) \\
&\qquad\qquad + (1 - \rho_\zeta) N_{\gamma \cap \nu}(\zeta) \Big] \\
&= (T_{\text{pre}} T_{\text{post}})^{-2} \left(N_\gamma^{\text{ctrl}} N_\nu^{\text{ctrl}}\right)^{-1} g_1(T_{\text{pre}}, T_{\text{post}}, \Delta) \\
&\quad \times \sum_{\zeta \in \Xi_{\text{ctrl}}} \left[(\phi_\zeta - \psi_\zeta) \left(\left(N_{\gamma \setminus \nu}(\zeta) + N_{\gamma \cap \nu}(\zeta)\right) N_\nu(\zeta) - N_{\gamma \cap \nu}(\zeta)\right) + (1 - \rho_\zeta) N_{\gamma \cap \nu}(\zeta) \right] \\
&= (T_{\text{pre}} T_{\text{post}})^{-2} \left(N_\gamma^{\text{ctrl}} N_\nu^{\text{ctrl}}\right)^{-1} g_1(T_{\text{pre}}, T_{\text{post}}, \Delta) \\
&\quad \times \sum_{\zeta \in \Xi_{\text{ctrl}}} \left[N_\gamma(\zeta) N_\nu(\zeta)(\phi_\zeta - \psi_\zeta) - N_{\gamma \cap \nu}(\zeta) \left((1 - \rho_\zeta) - (\phi_\zeta - \psi_\zeta)\right) \right].
\end{aligned}
$$

Substituting the definition of $g_1(T_{\text{pre}}, T_{\text{post}}, \Delta)$, we have

$$
\boxed{
\begin{aligned}
\text{Cov}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right) &= (T_{\text{pre}} T_{\text{post}})^{-2} \left(N_\gamma^{\text{ctrl}} N_\nu^{\text{ctrl}}\right)^{-1} \left(T_{\text{post}}^2 \left(T_{\text{pre}} - \Delta\right)_+ + T_{\text{pre}}^2 \left(T_{\text{post}} - \Delta\right)_+ - T_{\text{pre}} T_{\text{post}} \min(\cdots)\right) \\
&\quad \times \sum_{\zeta \in \Xi_{\text{ctrl}}} \left[N_\gamma(\zeta) N_\nu(\zeta)(\phi_\zeta - \psi_\zeta) - N_{\gamma \cap \nu}(\zeta) \left((1 - \rho_\zeta) - (\phi_\zeta - \psi_\zeta)\right) \right].
\end{aligned}
} \qquad (44)
$$

B Additional Simulation Results

We reproduce Table 1 in the main manuscript, this time with 10 control states (vs. 3). Interpretation of results remains unchanged, though because the expected correlations are smaller, the improvements in coverage and standard error from $\widehat{\text{ATT}}_{\text{gls}}$ over $\widehat{\text{ATT}}_{\text{ivw}}$ are minimal.

							$\text{Cor}\left(\widehat{\text{ATT}}_\gamma, \widehat{\text{ATT}}_\nu\right)$		$\widehat{\text{ATT}}_{\text{ivw}}$			$\widehat{\text{ATT}}_{\text{gls}}$		
T_{pre}	T_{post}	Δ	% Shared	ρ	ϕ	ψ	True Cor.	Est. Bias	Bias	SE	95% Covg.	Bias	SE	95% Covg.
1	1	1	0.25	0.10	0.06	0.02	-0.039	0.00	0.00	0.25	0.955	0.00	0.24	0.951
				0.60	0.40	0.20	-0.045	0.00	-0.01	1.05	0.955	-0.01	1.03	0.951
			0.75	0.10	0.06	0.02	-0.043	0.01	0.00	0.25	0.954	0.00	0.24	0.948
				0.60	0.40	0.20	-0.045	-0.01	0.01	1.05	0.954	0.01	1.03	0.949
		2	0.25	0.10	0.06	0.02	0.000	0.00	0.00	0.25	0.951	0.00	0.25	0.951
				0.60	0.40	0.20	0.000	-0.00	0.01	1.05	0.950	0.01	1.05	0.950
			0.75	0.10	0.06	0.02	0.000	-0.00	-0.00	0.25	0.952	-0.00	0.25	0.952
				0.60	0.40	0.20	0.000	0.01	-0.02	1.05	0.950	-0.02	1.05	0.950
5	5	3	0.25	0.10	0.06	0.02	0.008	0.01	0.00	0.11	0.949	0.00	0.11	0.949
				0.60	0.40	0.20	0.009	0.00	0.00	0.47	0.950	0.00	0.47	0.951
			0.75	0.10	0.06	0.02	0.009	0.01	0.00	0.11	0.948	0.00	0.11	0.949
				0.60	0.40	0.20	0.009	-0.01	-0.01	0.47	0.950	-0.01	0.47	0.952
		6	0.25	0.10	0.06	0.02	-0.032	-0.00	0.00	0.11	0.951	0.00	0.11	0.949
				0.60	0.40	0.20	-0.036	-0.00	-0.00	0.47	0.953	-0.00	0.46	0.949
			0.75	0.10	0.06	0.02	-0.035	-0.01	0.00	0.11	0.956	0.00	0.11	0.952
				0.60	0.40	0.20	-0.036	-0.00	-0.00	0.47	0.958	-0.00	0.46	0.952
7	3	3	0.25	0.10	0.06	0.02	-0.010	0.01	0.00	0.12	0.949	0.00	0.12	0.948
				0.60	0.40	0.20	-0.012	0.01	0.00	0.51	0.953	0.00	0.51	0.951
			0.75	0.10	0.06	0.02	-0.011	-0.02	0.00	0.12	0.950	0.00	0.12	0.949
				0.60	0.40	0.20	-0.012	-0.01	0.00	0.51	0.952	0.00	0.51	0.951
		6	0.25	0.10	0.06	0.02	-0.020	-0.00	0.00	0.12	0.955	0.00	0.12	0.953
				0.60	0.40	0.20	-0.023	-0.02	-0.00	0.51	0.957	-0.00	0.51	0.955
			0.75	0.10	0.06	0.02	-0.022	0.01	0.00	0.12	0.953	0.00	0.12	0.952
				0.60	0.40	0.20	-0.023	0.00	-0.01	0.51	0.953	-0.01	0.51	0.950

Table 9: Simulated between-estimate correlations along with standard error and 95% confidence interval coverage for aggregated estimates of $\widehat{\text{ATT}}$ for a variety of generative model parameters, 100 individuals per state, and 10 control states. Reported correlations are averages of 100 estimates generated from 100 simulations.

C Medical Cannabis Law Study

Here, we present sample sizes and other information needed to reproduce the correlation results presented in the main manuscript.

- **Treated States:** AR, CT, FL, LA, MD, MN, ND, NH, NY, OH, OK, PA
- **Control States:** AL, GA, ID, IN, IA, KS, KY, MS, NE, NC, SC, SD, TN, TX, VA, WI, WY

C.1 Total Counts Per Analysis

Cohort	N Treated	N Control	Total
AR	2,788	202,282	205,070
CT	1,919	111,339	113,258
FL	45,816	139,519	185,335
LA	6,253	207,053	213,306
MD	12,047	159,088	171,135
MN	23,216	121,426	144,642
ND	5,005	235,505	240,510
NH	1,241	134,254	135,495
NY	26,010	140,642	166,652
OH	19,907	235,544	255,451
OK	3,028	176,845	179,873
PA	6,144	171,175	177,319

Table 10: Sample sizes for each treated state's cohort, including the numbers of treated and control individuals. Using the notation from the manuscript, the number of treated individuals is $N_\gamma^{\text{tx}} := N_\gamma(\gamma)$, the number of control individuals is N_γ^{ctrl}, and the total sample size is N_γ, for $\gamma =$ AR, CT, ...

	AR	CT	FL	LA	MD	MN	ND	NH	NY	OH	OK	PA
AL	7518	5665	7006	7564	7438	6019	8565	6868	6882	8563	8022	7942
GA	48054	10180	12120	49047	13792	10502	49190	11896	12091	49102	14529	14062
ID	2555	1439	2068	2583	2301	1702	2954	2000	2006	2962	2680	2609
IN	13489	6179	9225	13650	11666	7629	15207	9069	9277	15193	14089	13434
IA	9595	6357	8112	9642	9334	7107	10464	7959	8018	10464	10146	9855
KS	3574	1910	2638	3614	2850	2112	3613	2541	2514	3641	3137	3030
KY	9295	2011	2588	12366	2805	2119	13503	2534	2615	13407	3049	2966
MS	2251	1284	1830	2250	2035	1389	2293	1814	1819	2289	2506	2404
NE	4255	3589	7087	4286	7515	3814	4600	4637	4816	4564	4727	4637
NC	20656	14726	19416	20608	21391	16994	25402	19035	19434	25419	23413	23033
SC	2634	1521	1825	2687	1951	1628	2773	1773	1903	2793	2378	2246
SD	1551	490	2087	1601	3291	1019	5368	1916	1732	5332	4399	3885
TN	8933	6557	7737	8976	9105	6826	10388	7635	7675	10407	10052	9646
TX	27937	23093	21978	27924	23071	23150	29395	21647	26043	29553	28273	27661
VA	9919	6189	8622	9962	9764	7394	11126	8398	8729	11162	10149	9935
WI	29471	19847	24755	29696	30241	21656	40049	24106	24676	40067	34674	33196
WY	595	302	425	597	538	366	615	426	412	626	622	634

Table 11: Number of individuals in each control state, by cohort

C.2 Counts of Disjoint Control Individuals

Ctrl. State	CT	FL	LA	MD	MN	ND	NH	NY	OH	OK	PA
AL	5,737	4,286	497	3,272	5,355	342	4,492	4,759	495	1,594	2,331
GA	45,578	43,057	2,606	40,749	45,027	2,273	43,427	43,959	3,170	36,927	38,951
IA	7,798	5,884	619	4,177	7,253	461	6,144	6,579	626	1,626	2,908
ID	2,152	1,652	165	1,279	1,983	123	1,723	1,804	163	583	863
IN	11,927	9,711	907	7,169	11,191	653	9,964	10,304	949	3,001	4,858
KS	3,085	2,448	255	2,002	2,938	204	2,552	2,687	280	1,141	1,553
KY	8,900	8,389	551	8,051	8,809	350	8,457	8,550	544	7,307	7,671
MS	2,002	1,568	171	1,211	1,947	162	1,616	1,698	225	510	888
NC	16,873	12,957	1,664	10,108	15,630	1,059	13,468	14,198	1,477	4,950	7,309
NE	3,567	2,663	282	2,082	3,382	245	2,769	2,930	383	1,125	1,620
SC	2,381	1,989	181	1,738	2,274	158	2,035	2,098	209	1,007	1,338
SD	1,439	1,208	324	940	1,370	316	1,244	1,278	338	540	719
TN	7,258	5,683	655	4,265	6,782	472	5,848	6,141	649	1,957	3,142
TX	24,646	20,712	2,414	17,138	23,569	1,635	21,241	22,010	2,227	6,721	10,655
VA	8,197	6,172	698	4,717	7,584	529	6,451	6,814	714	2,339	3,506
WI	24,902	19,775	2,303	14,551	23,279	1,761	20,466	21,445	2,300	6,659	10,329
WY	526	426	41	298	503	32	434	466	42	105	178

Table 12: Counts of disjoint control individuals in Arkansas cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Arkansas and not the other, paired treated state (column).

Ctrl. State	AR	FL	LA	MD	MN	ND	NH	NY	OH	OK	PA
AL	3,884	1,949	3,946	2,622	1,071	3,520	1,816	1,493	3,485	3,326	2,986
GA	7,704	4,382	7,784	5,680	2,477	7,556	4,120	3,379	7,517	7,079	6,527
IA	4,560	2,696	4,619	3,449	1,381	4,358	2,502	1,965	4,335	4,176	3,874
ID	1,036	582	1,052	757	319	966	541	451	964	929	856
IN	4,617	2,656	4,672	3,355	1,429	4,357	2,493	1,943	4,322	4,099	3,762
KS	1,421	832	1,441	1,065	489	1,414	765	656	1,398	1,278	1,197
KY	1,616	867	1,628	1,066	517	1,592	809	670	1,579	1,300	1,207
MS	1,035	585	1,039	765	269	1,027	538	429	1,013	919	861
NC	10,943	5,721	11,118	7,498	3,107	9,831	5,373	4,256	9,769	9,214	8,454
NE	2,901	1,583	2,923	2,055	808	2,794	1,480	1,123	2,783	2,611	2,434
SC	1,268	758	1,279	944	419	1,254	730	562	1,251	1,098	1,029
SD	378	203	385	252	121	308	191	152	307	302	282
TN	4,882	2,712	4,963	3,380	1,559	4,525	2,538	2,123	4,493	4,163	3,848
TX	19,802	13,691	19,956	16,491	7,070	19,512	13,144	9,048	19,451	18,882	18,037
VA	4,467	2,677	4,529	3,335	1,409	4,232	2,550	1,933	4,222	4,073	3,752
WI	15,278	8,769	15,435	10,935	5,176	13,926	8,331	6,761	13,832	13,161	12,206
WY	233	124	234	169	63	230	109	91	225	209	194

Table 13: Counts of disjoint control individuals in Connecticut cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Connecticut and not the other, paired treated state (column).

Ctrl. State	AR	CT	LA	MD	MN	ND	NH	NY	OH	OK	PA
AL	3,774	3,290	3,904	1,384	2,319	3,151	579	1,183	3,085	2,731	2,143
GA	7,123	6,322	7,344	2,860	4,540	6,708	1,180	2,412	6,587	5,731	4,637
IA	4,401	4,451	4,573	1,726	2,967	3,880	787	1,650	3,822	3,413	2,710
ID	1,165	1,211	1,210	426	791	974	203	396	955	851	672
IN	5,447	5,702	5,620	2,240	3,716	4,813	852	1,801	4,729	4,190	3,402
KS	1,512	1,560	1,566	610	1,086	1,453	305	596	1,416	1,186	946
KY	1,682	1,444	1,722	576	1,064	1,617	260	530	1,595	1,155	916
MS	1,147	1,131	1,168	485	939	1,116	210	440	1,090	923	728
NC	11,717	10,411	12,140	4,118	6,678	9,404	1,753	3,516	9,256	8,000	6,401
NE	5,495	5,081	5,552	1,510	4,416	5,264	2,816	3,282	5,224	4,756	4,353
SC	1,180	1,062	1,203	457	743	1,115	186	363	1,095	845	669
SD	1,744	1,800	1,758	364	1,324	828	283	644	818	745	605
TN	4,487	3,892	4,658	1,489	2,644	3,762	674	1,370	3,695	3,071	2,449
TX	14,753	12,576	15,154	6,796	8,541	13,985	2,171	4,492	13,784	12,460	10,507
VA	4,875	5,110	5,040	1,842	3,270	4,236	787	1,654	4,166	3,733	3,013
WI	15,059	13,677	15,449	5,076	8,832	11,810	2,358	4,702	11,538	10,025	7,980
WY	256	247	264	104	156	250	35	93	241	207	150

Table 14: Counts of disjoint control individuals in Florida cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Florida and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	MD	MN	ND	NH	NY	OH	OK	PA
AL	543	5,845	4,462	3,476	5,502	781	4,678	4,927	900	1,872	2,624
GA	3,599	46,651	44,271	42,090	46,174	5,291	44,685	45,199	6,067	38,459	40,475
IA	666	7,904	6,103	4,462	7,402	1,014	6,367	6,784	1,152	2,040	3,313
ID	193	2,196	1,725	1,359	2,038	285	1,798	1,873	314	698	976
IN	1,068	12,143	10,045	7,598	11,458	1,494	10,303	10,631	1,733	3,624	5,457
KS	295	3,145	2,542	2,128	3,015	464	2,653	2,785	522	1,311	1,714
KY	3,622	11,983	11,500	11,177	11,904	1,165	11,567	11,654	1,359	10,476	10,828
MS	170	2,005	1,588	1,249	1,957	276	1,647	1,725	326	568	951
NC	1,616	17,000	13,332	10,616	15,887	2,323	13,894	14,561	2,648	5,775	8,083
NE	313	3,620	2,751	2,197	3,463	486	2,876	3,029	599	1,280	1,783
SC	234	2,445	2,065	1,825	2,350	333	2,126	2,184	379	1,141	1,474
SD	374	1,496	1,272	1,012	1,433	425	1,312	1,344	445	632	804
TN	698	7,382	5,897	4,551	6,951	996	6,089	6,366	1,137	2,365	3,541
TX	2,401	24,787	21,100	17,699	23,836	3,583	21,670	22,388	4,058	8,031	11,885
VA	741	8,302	6,380	4,985	7,730	1,142	6,683	7,031	1,281	2,742	3,889
WI	2,528	25,284	20,390	15,378	23,781	3,587	21,106	22,031	4,030	7,852	11,532
WY	43	529	436	313	507	60	442	474	69	124	202

Table 15: Counts of disjoint control individuals in Louisiana cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Louisiana and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	LA	MN	ND	NH	NY	OH	OK	PA
AL	3,192	4,395	1,816	3,350	3,608	2,423	2,192	2,668	2,336	1,900	1,119
GA	6,487	9,292	4,532	6,835	7,932	5,920	5,273	6,238	5,777	4,538	2,856
IA	3,916	6,426	2,948	4,154	5,227	3,202	3,438	4,161	3,117	2,556	1,546
ID	1,025	1,619	659	1,077	1,271	768	800	951	745	613	366
IN	5,346	8,842	4,681	5,614	7,312	4,277	5,164	5,837	4,131	3,320	2,053
KS	1,278	2,005	822	1,364	1,644	1,198	1,008	1,248	1,155	869	515
KY	1,561	1,860	793	1,616	1,571	1,482	948	1,143	1,455	881	555
MS	995	1,516	690	1,034	1,369	953	809	978	914	700	398
NC	10,843	14,163	6,093	11,399	11,211	7,754	7,212	8,636	7,542	5,863	3,592
NE	5,342	5,981	1,938	5,426	5,439	5,058	4,138	4,525	5,011	4,352	3,819
SC	1,055	1,374	583	1,089	1,129	961	704	841	935	620	365
SD	2,680	3,053	1,568	2,702	2,662	919	1,781	2,082	896	774	499
TN	4,437	5,928	2,857	4,680	4,886	3,446	3,265	3,857	3,355	2,522	1,548
TX	12,272	16,469	7,889	12,846	13,705	11,138	9,087	10,774	10,884	8,904	5,985
VA	4,562	6,910	2,984	4,787	5,440	3,684	3,486	4,155	3,588	3,002	1,974
WI	15,321	21,329	10,562	15,923	17,401	10,429	12,076	13,969	10,081	7,917	4,774
WY	241	405	217	254	335	228	237	286	219	174	84

Table 16: Counts of disjoint control individuals in Maryland cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Maryland and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	LA	MD	ND	NH	NY	OH	OK	PA
AL	3,856	1,425	1,332	3,957	2,189	3,396	1,144	746	3,345	3,125	2,695
GA	7,475	2,799	2,922	7,629	4,642	7,229	2,514	1,508	7,135	6,527	5,797
IA	4,765	2,131	1,962	4,867	3,000	4,411	1,693	997	4,371	4,116	3,646
ID	1,130	582	425	1,157	672	1,021	368	236	1,010	952	834
IN	5,331	2,879	2,120	5,437	3,275	4,924	1,873	1,065	4,868	4,542	4,014
KS	1,476	691	560	1,513	906	1,445	472	324	1,414	1,249	1,132
KY	1,633	625	595	1,657	885	1,597	497	283	1,576	1,239	1,097
MS	1,085	374	498	1,096	723	1,069	415	276	1,052	947	857
NC	11,968	5,375	4,256	12,273	6,814	10,346	3,713	2,205	10,233	9,414	8,283
NE	2,941	1,033	1,143	2,991	1,738	2,797	999	567	2,775	2,542	2,283
SC	1,268	526	546	1,291	806	1,240	500	266	1,229	1,048	948
SD	838	650	256	851	390	557	220	122	555	531	477
TN	4,675	1,828	1,733	4,801	2,607	4,176	1,490	949	4,128	3,696	3,266
TX	18,782	7,127	9,713	19,062	13,784	18,273	8,882	3,581	18,149	17,298	16,058
VA	5,059	2,614	2,042	5,162	3,070	4,619	1,836	1,051	4,582	4,305	3,821
WI	15,464	6,985	5,733	15,741	8,816	13,335	5,064	2,959	13,146	12,155	10,784
WY	274	127	97	276	163	268	80	48	261	237	199

Table 17: Counts of disjoint control individuals in Minnesota cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Minnesota and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	LA	MD	MN	NH	NY	OH	OK	PA
AL	1,389	6,420	4,710	1,782	3,550	5,942	4,957	5,239	209	1,505	2,395
GA	3,409	46,566	43,778	5,434	41,318	45,917	44,211	44,739	1,153	37,138	39,262
IA	1,330	8,465	6,232	1,836	4,332	7,768	6,528	6,991	237	1,436	2,827
ID	522	2,481	1,860	656	1,421	2,273	1,949	2,039	57	579	910
IN	2,371	13,385	10,795	3,051	7,818	12,502	11,099	11,462	394	2,921	5,002
KS	243	3,117	2,428	463	1,961	2,946	2,539	2,674	101	1,038	1,480
KY	4,558	13,084	12,532	2,302	12,180	12,981	12,612	12,703	313	11,356	11,737
MS	204	2,036	1,579	319	1,211	1,973	1,638	1,711	93	458	832
NC	5,805	20,507	15,390	7,117	11,765	18,754	16,073	16,946	655	5,313	8,141
NE	590	3,805	2,777	800	2,143	3,583	2,905	3,068	179	1,050	1,584
SC	297	2,506	2,063	419	1,783	2,385	2,121	2,189	62	971	1,331
SD	4,133	5,186	4,109	4,192	2,996	4,906	4,263	4,489	148	1,304	2,110
TN	1,927	8,356	6,413	2,408	4,729	7,738	6,620	6,971	248	1,932	3,309
TX	3,093	25,814	21,402	5,054	17,462	24,518	22,006	22,787	815	5,972	10,185
VA	1,736	9,169	6,740	2,306	5,046	8,351	7,058	7,450	275	2,269	3,582
WI	12,339	34,128	27,104	13,940	20,237	31,728	28,013	29,219	957	9,764	14,559
WY	52	543	440	78	305	517	450	480	15	95	179

Table 18: Counts of disjoint control individuals in North Dakota cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for North Dakota and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	LA	MD	MN	ND	NY	OH	OK	PA
AL	3,842	3,019	441	3,982	1,622	1,993	3,260	768	3,200	2,881	2,303
GA	7,269	5,836	956	7,534	3,377	3,908	6,917	1,556	6,807	6,023	4,976
IA	4,508	4,104	634	4,684	2,063	2,545	4,023	1,059	3,970	3,625	2,942
ID	1,168	1,102	135	1,215	499	666	995	236	979	889	717
IN	5,544	5,383	696	5,722	2,567	3,313	4,961	1,192	4,887	4,399	3,630
KS	1,519	1,396	208	1,580	699	901	1,467	345	1,429	1,211	1,003
KY	1,696	1,332	206	1,735	677	912	1,643	322	1,617	1,212	990
MS	1,179	1,068	194	1,211	588	840	1,159	285	1,137	993	801
NC	11,847	9,682	1,372	12,321	4,856	5,754	9,706	2,242	9,577	8,452	6,902
NE	3,151	2,528	366	3,227	1,260	1,822	2,942	591	2,910	2,476	2,088
SC	1,174	982	134	1,212	526	645	1,121	227	1,103	874	718
SD	1,609	1,617	112	1,627	406	1,117	811	403	802	745	607
TN	4,550	3,616	572	4,748	1,795	2,299	3,867	891	3,815	3,247	2,656
TX	14,951	11,698	1,840	15,393	7,663	7,379	14,258	2,924	14,087	12,912	11,024
VA	4,930	4,759	563	5,119	2,120	2,840	4,330	1,057	4,269	3,873	3,175
WI	15,101	12,590	1,709	15,516	5,941	7,514	12,070	2,912	11,828	10,494	8,538
WY	265	233	36	271	125	140	261	66	252	222	169

Table 19: Counts of disjoint control individuals in New Hampshire cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for New Hampshire and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	LA	MD	MN	ND	NH	OH	OK	PA
AL	4,123	2,710	1,059	4,245	2,112	1,609	3,556	782	3,501	3,220	2,674
GA	7,996	5,290	2,383	8,243	4,537	3,097	7,640	1,751	7,533	6,831	5,845
IA	5,002	3,626	1,556	5,160	2,845	1,908	4,545	1,118	4,486	4,205	3,541
ID	1,255	1,018	334	1,296	656	540	1,091	242	1,075	994	835
IN	6,092	5,041	1,853	6,258	3,448	2,713	5,532	1,400	5,457	5,028	4,305
KS	1,627	1,260	472	1,685	912	726	1,575	318	1,537	1,339	1,147
KY	1,870	1,274	557	1,903	953	779	1,815	403	1,793	1,406	1,204
MS	1,266	964	429	1,294	762	706	1,237	290	1,215	1,083	913
NC	12,976	8,964	3,534	13,387	6,679	4,645	10,978	2,641	10,833	9,833	8,339
NE	3,491	2,350	1,011	3,559	1,826	1,569	3,284	770	3,252	2,870	2,513
SC	1,367	944	441	1,400	793	541	1,319	357	1,306	1,092	953
SD	1,459	1,394	289	1,475	523	835	853	219	845	800	685
TN	4,883	3,241	1,308	5,065	2,427	1,798	4,258	931	4,204	3,697	3,144
TX	20,116	11,998	8,557	20,507	13,746	6,474	19,435	7,320	19,273	18,230	16,506
VA	5,624	4,473	1,761	5,798	3,120	2,386	5,053	1,388	4,993	4,655	3,984
WI	16,650	11,590	4,623	17,011	8,404	5,979	13,846	3,482	13,610	12,449	10,547
WY	283	201	80	289	160	94	277	52	267	241	194

Table 20: Counts of disjoint control individuals in New York cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for New York and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	LA	MD	MN	ND	NH	NY	OK	PA
AL	1,540	6,383	4,642	1,899	3,461	5,889	207	4,895	5,182	1,362	2,246
GA	4,218	46,439	43,569	6,122	41,087	45,735	1,065	44,013	44,544	36,774	38,921
IA	1,495	8,442	6,174	1,974	4,247	7,728	237	6,475	6,932	1,282	2,678
ID	570	2,487	1,849	693	1,406	2,270	65	1,941	2,031	538	875
IN	2,653	13,336	10,697	3,276	7,658	12,432	380	11,011	11,373	2,656	4,747
KS	347	3,129	2,419	549	1,946	2,943	129	2,529	2,664	996	1,436
KY	4,656	12,975	12,414	2,400	12,057	12,864	217	12,490	12,585	11,216	11,595
MS	263	2,018	1,549	365	1,168	1,952	89	1,612	1,685	400	774
NC	6,240	20,462	15,259	7,459	11,570	18,658	672	15,961	16,818	4,917	7,741
NE	692	3,758	2,701	877	2,060	3,525	143	2,837	3,000	933	1,475
SC	368	2,523	2,063	485	1,777	2,394	82	2,123	2,196	941	1,301
SD	4,119	5,149	4,063	4,176	2,937	4,868	112	4,218	4,445	1,194	2,021
TN	2,123	8,343	6,365	2,568	4,657	7,709	267	6,587	6,936	1,766	3,168
TX	3,843	25,911	21,359	5,687	17,366	24,552	973	21,993	22,783	5,505	9,764
VA	1,957	9,195	6,706	2,481	4,986	8,350	311	7,033	7,426	2,113	3,432
WI	12,896	34,052	26,850	14,401	19,907	31,557	975	27,789	29,001	9,166	14,003
WY	73	549	442	98	307	521	26	452	481	86	169

Table 21: Counts of disjoint control individuals in Ohio cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Ohio and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	LA	MD	MN	ND	NH	NY	OH	PA
AL	2,098	5,683	3,747	2,330	2,484	5,128	962	4,035	4,360	821	1,244
GA	3,402	11,428	8,140	3,941	5,275	10,554	2,477	8,656	9,269	2,201	3,000
IA	2,177	7,965	5,447	2,544	3,368	7,155	1,118	5,812	6,333	964	1,801
ID	708	2,170	1,463	795	992	1,930	305	1,569	1,668	256	453
IN	3,601	12,009	9,054	4,063	5,743	11,002	1,803	9,419	9,840	1,552	2,731
KS	704	2,505	1,685	834	1,156	2,274	562	1,807	1,962	492	603
KY	1,061	2,338	1,616	1,159	1,125	2,169	902	1,727	1,840	858	596
MS	765	2,141	1,599	824	1,171	2,064	671	1,685	1,770	617	587
NC	7,707	17,901	11,997	8,580	7,885	15,833	3,324	12,830	13,812	2,911	3,976
NE	1,597	3,749	2,396	1,721	1,564	3,455	1,177	2,566	2,781	1,096	852
SC	751	1,955	1,398	832	1,047	1,798	576	1,479	1,567	526	535
SD	3,388	4,211	3,057	3,430	1,882	3,911	335	3,228	3,467	261	975
TN	3,076	7,658	5,386	3,441	3,469	6,922	1,596	5,664	6,074	1,411	1,918
TX	7,057	24,062	18,755	8,380	14,106	22,421	4,850	19,538	20,460	4,225	5,914
VA	2,569	8,033	5,260	2,929	3,387	7,060	1,292	5,624	6,075	1,100	1,826
WI	11,862	27,988	19,944	12,830	12,350	25,173	4,389	21,062	22,447	3,773	6,343
WY	132	529	404	149	258	493	102	418	451	82	120

Table 22: Counts of disjoint control individuals in Oklahoma cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Oklahoma and not the other, paired treated state (column).

Ctrl. State	AR	CT	FL	LA	MD	MN	ND	NH	NY	OH	OK
AL	2,755	5,263	3,079	3,002	1,623	4,618	1,772	3,377	3,734	1,625	1,164
GA	4,959	10,409	6,579	5,490	3,126	9,357	4,134	7,142	7,816	3,881	2,533
IA	3,168	7,372	4,453	3,526	2,067	6,394	2,218	4,838	5,378	2,069	1,510
ID	917	2,026	1,213	1,002	674	1,741	565	1,326	1,438	522	382
IN	4,803	11,017	7,611	5,241	3,821	9,819	3,229	7,995	8,462	2,988	2,076
KS	1,009	2,317	1,338	1,130	695	2,050	897	1,492	1,663	825	496
KY	1,342	2,162	1,294	1,428	716	1,944	1,200	1,422	1,555	1,154	513
MS	1,041	1,981	1,302	1,105	767	1,872	943	1,391	1,498	889	485
NC	9,686	16,761	10,018	10,508	5,234	14,322	5,772	10,900	11,938	5,355	3,596
NE	2,002	3,482	1,903	2,134	941	3,106	1,621	2,088	2,334	1,548	762
SC	950	1,754	1,090	1,033	660	1,566	804	1,191	1,296	754	403
SD	3,053	3,677	2,403	3,088	1,093	3,343	627	2,576	2,838	574	461
TN	3,855	6,937	4,358	4,211	2,089	6,086	2,567	4,667	5,115	2,407	1,512
TX	10,379	22,605	16,190	11,622	10,575	20,569	8,451	17,038	18,124	7,872	5,302
VA	3,522	7,498	4,326	3,862	2,145	6,362	2,391	4,712	5,190	2,205	1,612
WI	14,054	25,555	16,421	15,032	7,729	22,324	7,706	17,628	19,067	7,132	4,865
WY	217	526	359	239	180	467	198	377	416	177	132

Table 23: Counts of disjoint control individuals in Pennsylvania cohort, by paired analysis. Each entry is the number of individuals in a control state (row) who contribute to only the analysis for Pennsylvania and not the other, paired treated state (column).

C.3 Counts of Shared Control Individuals

Ctrl. State	CT	FL	LA	MD	MN	ND	NH	NY	OH	OK	PA
AL	1,781	3,232	7,021	4,246	2,163	7,176	3,026	2,759	7,023	5,924	5,187
GA	2,476	4,997	45,448	7,305	3,027	45,781	4,627	4,095	44,884	11,127	9,103
IA	1,797	3,711	8,976	5,418	2,342	9,134	3,451	3,016	8,969	7,969	6,687
ID	403	903	2,390	1,276	572	2,432	832	751	2,392	1,972	1,692
IN	1,562	3,778	12,582	6,320	2,298	12,836	3,525	3,185	12,540	10,488	8,631
KS	489	1,126	3,319	1,572	636	3,370	1,022	887	3,294	2,433	2,021
KY	395	906	8,744	1,244	486	8,945	838	745	8,751	1,988	1,624
MS	249	683	2,080	1,040	304	2,089	635	553	2,026	1,741	1,363
NC	3,783	7,699	18,992	10,548	5,026	19,597	7,188	6,458	19,179	15,706	13,347
NE	688	1,592	3,973	2,173	873	4,010	1,486	1,325	3,872	3,130	2,635
SC	253	645	2,453	896	360	2,476	599	536	2,425	1,627	1,296
SD	112	343	1,227	611	181	1,235	307	273	1,213	1,011	832
TN	1,675	3,250	8,278	4,668	2,151	8,461	3,085	2,792	8,284	6,976	5,791
TX	3,291	7,225	25,523	10,799	4,368	26,302	6,696	5,927	25,710	21,216	17,282
VA	1,722	3,747	9,221	5,202	2,335	9,390	3,468	3,105	9,205	7,580	6,413
WI	4,569	9,696	27,168	14,920	6,192	27,710	9,005	8,026	27,171	22,812	19,142
WY	69	169	554	297	92	563	161	129	553	490	417

Table 24: Counts of control individuals shared between Arkansas and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for Arkansas and the analysis for the other, paired treated state (column).

Ctrl. State	FL	LA	MD	MN	ND	NH	NY	OH	OK	PA
AL	3,716	1,719	3,043	4,594	2,145	3,849	4,172	2,180	2,339	2,679
GA	5,798	2,396	4,500	7,703	2,624	6,060	6,801	2,663	3,101	3,653
IA	3,661	1,738	2,908	4,976	1,999	3,855	4,392	2,022	2,181	2,483
ID	857	387	682	1,120	473	898	988	475	510	583
IN	3,523	1,507	2,824	4,750	1,822	3,686	4,236	1,857	2,080	2,417
KS	1,078	469	845	1,421	496	1,145	1,254	512	632	713
KY	1,144	383	945	1,494	419	1,202	1,341	432	711	804
MS	699	245	519	1,015	257	746	855	271	365	423
NC	9,005	3,608	7,228	11,619	4,895	9,353	10,470	4,957	5,512	6,272
NE	2,006	666	1,534	2,781	795	2,109	2,466	806	978	1,155
SC	763	242	577	1,102	267	791	959	270	423	492
SD	287	105	238	369	182	299	338	183	188	208
TN	3,845	1,594	3,177	4,998	2,032	4,019	4,434	2,064	2,394	2,709
TX	9,402	3,137	6,602	16,023	3,581	9,949	14,045	3,642	4,211	5,056
VA	3,512	1,660	2,854	4,780	1,957	3,639	4,256	1,967	2,116	2,437
WI	11,078	4,412	8,912	14,671	5,921	11,516	13,086	6,015	6,686	7,641
WY	178	68	133	239	72	193	211	77	93	108

Table 25: Counts of control individuals shared between Connecticut and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for Connecticut and the analysis for the other, paired treated state (column).

Ctrl. State	LA	MD	MN	ND	NH	NY	OH	OK	PA
AL	3,102	5,622	4,687	3,855	6,427	5,823	3,921	4,275	4,863
GA	4,776	9,260	7,580	5,412	10,940	9,708	5,533	6,389	7,483
IA	3,539	6,386	5,145	4,232	7,325	6,462	4,290	4,699	5,402
ID	858	1,642	1,277	1,094	1,865	1,672	1,113	1,217	1,396
IN	3,605	6,985	5,509	4,412	8,373	7,424	4,496	5,035	5,823
KS	1,072	2,028	1,552	1,185	2,333	2,042	1,222	1,452	1,692
KY	866	2,012	1,524	971	2,328	2,058	993	1,433	1,672
MS	662	1,345	891	714	1,620	1,390	740	907	1,102
NC	7,276	15,298	12,738	10,012	17,663	15,900	10,160	11,416	13,015
NE	1,535	5,577	2,671	1,823	4,271	3,805	1,863	2,331	2,734
SC	622	1,368	1,082	710	1,639	1,462	730	980	1,156
SD	329	1,723	763	1,259	1,804	1,443	1,269	1,342	1,482
TN	3,079	6,248	5,093	3,975	7,063	6,367	4,042	4,666	5,288
TX	6,824	15,182	13,437	7,993	19,807	17,486	8,194	9,518	11,471
VA	3,582	6,780	5,352	4,386	7,835	6,968	4,456	4,889	5,609
WI	9,306	19,679	15,923	12,945	22,397	20,053	13,217	14,730	16,775
WY	161	321	269	175	390	332	184	218	275

Table 26: Counts of control individuals shared between Florida and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for Florida and the analysis for the other, paired treated state (column).

Ctrl. State	MD	MN	ND	NH	NY	OH	OK	PA
AL	4,088	2,062	6,783	2,886	2,637	6,664	5,692	4,940
GA	6,957	2,873	43,756	4,362	3,848	42,980	10,588	8,572
IA	5,180	2,240	8,628	3,275	2,858	8,490	7,602	6,329
ID	1,224	545	2,298	785	710	2,269	1,885	1,607
IN	6,052	2,192	12,156	3,347	3,019	11,917	10,026	8,193
KS	1,486	599	3,150	961	829	3,092	2,303	1,900
KY	1,189	462	11,201	799	712	11,007	1,890	1,538
MS	1,001	293	1,974	603	525	1,924	1,682	1,299
NC	9,992	4,721	18,285	6,714	6,047	17,960	14,833	12,525
NE	2,089	823	3,800	1,410	1,257	3,687	3,006	2,503
SC	862	337	2,354	561	503	2,308	1,546	1,213
SD	589	168	1,176	289	257	1,156	969	797
TN	4,425	2,025	7,980	2,887	2,610	7,839	6,611	5,435
TX	10,225	4,088	24,341	6,254	5,536	23,866	19,893	16,039
VA	4,977	2,232	8,820	3,279	2,931	8,681	7,220	6,073
WI	14,318	5,915	26,109	8,590	7,665	25,666	21,844	18,164
WY	284	90	537	155	123	528	473	395

Table 27: Counts of control individuals shared between Louisiana and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for Louisiana and the analysis for the other, paired treated state (column).

Ctrl. State	MN	ND	NH	NY	OH	OK	PA
AL	3,830	5,015	5,246	4,770	5,102	5,538	6,319
GA	5,860	7,872	8,519	7,554	8,015	9,254	10,936
IA	4,107	6,132	5,896	5,173	6,217	6,778	7,788
ID	1,030	1,533	1,501	1,350	1,556	1,688	1,935
IN	4,354	7,389	6,502	5,829	7,535	8,346	9,613
KS	1,206	1,652	1,842	1,602	1,695	1,981	2,335
KY	1,234	1,323	1,857	1,662	1,350	1,924	2,250
MS	666	1,082	1,226	1,057	1,121	1,335	1,637
NC	10,180	13,637	14,179	12,755	13,849	15,528	17,799
NE	2,076	2,457	3,377	2,990	2,504	3,163	3,696
SC	822	990	1,247	1,110	1,016	1,331	1,586
SD	629	2,372	1,510	1,209	2,395	2,517	2,792
TN	4,219	5,659	5,840	5,248	5,750	6,583	7,557
TX	9,366	11,933	13,984	12,297	12,187	14,167	17,086
VA	4,324	6,080	6,278	5,609	6,176	6,762	7,790
WI	12,840	19,812	18,165	16,272	20,160	22,324	25,467
WY	203	310	301	252	319	364	454

Table 28: Counts of control individuals shared between Maryland and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for Maryland and the analysis for the other, paired treated state (column).

Ctrl. State	ND	NH	NY	OH	OK	PA
AL	2,623	4,875	5,273	2,674	2,894	3,324
GA	3,273	7,988	8,994	3,367	3,975	4,705
IA	2,696	5,414	6,110	2,736	2,991	3,461
ID	681	1,334	1,466	692	750	868
IN	2,705	5,756	6,564	2,761	3,087	3,615
KS	667	1,640	1,788	698	863	980
KY	522	1,622	1,836	543	880	1,022
MS	320	974	1,113	337	442	532
NC	6,648	13,281	14,789	6,761	7,580	8,711
NE	1,017	2,815	3,247	1,039	1,272	1,531
SC	388	1,128	1,362	399	580	680
SD	462	799	897	464	488	542
TN	2,650	5,336	5,877	2,698	3,130	3,560
TX	4,877	14,268	19,569	5,001	5,852	7,092
VA	2,775	5,558	6,343	2,812	3,089	3,573
WI	8,321	16,592	18,697	8,510	9,501	10,872
WY	98	286	318	105	129	167

Table 29: Counts of control individuals shared between Minnesota and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for Minnesota and the analysis for the other, paired treated state (column).

Ctrl. State	NH	NY	OH	OK	PA
AL	3,608	3,326	8,356	7,060	6,170
GA	4,979	4,451	48,037	12,052	9,928
IA	3,936	3,473	10,227	9,028	7,637
ID	1,005	915	2,897	2,375	2,044
IN	4,108	3,745	14,813	12,286	10,205
KS	1,074	939	3,512	2,575	2,133
KY	891	800	13,190	2,147	1,766
MS	655	582	2,200	1,835	1,461
NC	9,329	8,456	24,747	20,089	17,261
NE	1,695	1,532	4,421	3,550	3,016
SC	652	584	2,711	1,802	1,442
SD	1,105	879	5,220	4,064	3,258
TN	3,768	3,417	10,140	8,456	7,079
TX	7,389	6,608	28,580	23,423	19,210
VA	4,068	3,676	10,851	8,857	7,544
WI	12,036	10,830	39,092	30,285	25,490
WY	165	135	600	520	436

Table 30: Counts of control individuals shared between North Dakota and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for North Dakota and the analysis for the other, paired treated state (column).

Ctrl. State	NY	OH	OK	PA
AL	6,100	3,668	3,987	4,565
GA	10,340	5,089	5,873	6,920
IA	6,900	3,989	4,334	5,017
ID	1,764	1,021	1,111	1,283
IN	7,877	4,182	4,670	5,439
KS	2,196	1,112	1,330	1,538
KY	2,212	917	1,322	1,544
MS	1,529	677	821	1,013
NC	16,793	9,458	10,583	12,133
NE	4,046	1,727	2,161	2,549
SC	1,546	670	899	1,055
SD	1,513	1,114	1,171	1,309
TN	6,744	3,820	4,388	4,979
TX	18,723	7,560	8,735	10,623
VA	7,341	4,129	4,525	5,223
WI	21,194	12,278	13,612	15,568
WY	360	174	204	257

Table 31: Counts of control individuals shared between New Hampshire and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for New Hampshire and the analysis for the other, paired treated state (column).

Ctrl. State	OH	OK	PA
AL	3,381	3,662	4,208
GA	4,558	5,260	6,246
IA	3,532	3,813	4,477
ID	931	1,012	1,171
IN	3,820	4,249	4,972
KS	977	1,175	1,367
KY	822	1,209	1,411
MS	604	736	906
NC	8,601	9,601	11,095
NE	1,564	1,946	2,303
SC	597	811	950
SD	887	932	1,047
TN	3,471	3,978	4,531
TX	6,770	7,813	9,537
VA	3,736	4,074	4,745
WI	11,066	12,227	14,129
WY	145	171	218

Table 32: Counts of control individuals shared between New York and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for New York and the analysis for the other, paired treated state (column).

Ctrl. State	OK	PA
AL	7,201	6,317
GA	12,328	10,181
IA	9,182	7,786
ID	2,424	2,087
IN	12,537	10,446
KS	2,645	2,205
KY	2,191	1,812
MS	1,889	1,515
NC	20,502	17,678
NE	3,631	3,089
SC	1,852	1,492
SD	4,138	3,311
TN	8,641	7,239
TX	24,048	19,789
VA	9,049	7,730
WI	30,901	26,064
WY	540	457

Table 33: Counts of control individuals shared between Ohio and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for Ohio and the analysis for the other, paired treated state (column).

Ctrl. State	PA
AL	6,778
GA	11,529
IA	8,345
ID	2,227
IN	11,358
KS	2,534
KY	2,453
MS	1,919
NC	19,437
NE	3,875
SC	1,843
SD	3,424
TN	8,134
TX	22,359
VA	8,323
WI	28,331
WY	502

Table 34: Counts of control individuals shared between Oklahoma and each other cohort. Each entry is the number of individuals in a control state (row) who contribute to the analysis for Oklahoma and the analysis for the other, paired treated state (column).